## PECS11 - Probability Theory and Stochastic Processes

P. Pages: 2

Time : Three Hours

GUG/W/18/10978
Max. Marks : 70

Notes : 1. Attempt any five questions.
2. All questions carry equal marks.
3. Illustrate your answers wherever necessary with the help of neat sketches.

1. a) Define Distribution function and Briefly explain its properties.
b) A certain test for a particular cancer is known to be $95 \%$ accurate. A person submits to be the test and results are positive. Suppose that the person comes from a population of $1,00,000$ where 200 people suffer from that disease. What can we conclude about the probability that the person under the test has that particular cancer?
2. a) Determine binomial distribution for which mean is 4 and variance is 3 .
b) A fair coin is tossed twice and let random variable X represent number of heads. find $\mathrm{F}_{\mathrm{X}}(\mathrm{X})$.
3. a) A set of 8 symmetrical coins was tossed 256 times and the frequencies of throws observed were as follows.

| Number of heads | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Freq. of throws | 2 | 6 | 24 | 63 | 64 | 50 | 36 | 10 | 1 |

Fit a binomial distribution to the data and hence as otherwise calculate the theoretical frequencies.
4. a) Find the first four moments about mean for a random variable $X$ having density function.
$f(x)=\left\{\begin{array}{cc}\frac{4}{81} x\left(9-x^{2}\right) & 0 \leq x \leq 3 \\ 0 & \text { otherwise }\end{array}\right.$
b) Define continuous random variable. State its properties.
5. a) Let $X \sim U([\pi,-\pi])$. Find the distributions of random variable $y=\cos x$ the density function. X is given by.
$f_{X}(x)=\left\{\begin{array}{cc}1 / 2 \pi & \text { if } x \in(-\pi, \pi) \\ 0 & \text { otherwise }\end{array}\right.$
b) Explain Normal distribution with its properties.
6. a) Given $f(x)=\left\{\begin{array}{cc}x / 2 & \text { if } 0 \leq x \leq 2 \\ 0 & \text { otherwise }\end{array}\right.$
and $g(y)=\left\{\begin{array}{cc}2(1-y) & \text { if } 0 \leq y \leq 1 \\ 0 & \text { otherwise }\end{array}\right.$
Determine the function $y(x)$ which will transform $f(x)$ into $g(y)$.
7. a) Suppose $X(t)$ is normal process with $n(t)=3, c\left(t_{1}, t_{2}\right)=4 e^{-0.2\left|t_{1}-t_{2}\right|}$
a) Find the probability that $\mathrm{X}(5) \leq 2$
b) Find the probability that

$$
|x(8)-X(5)| \leq 1 .
$$

b) Establish necessary and sufficient condition for the process.
$\mathrm{X}(\mathrm{t})=\mathrm{a} \cos \mathrm{w} \mathrm{t}+\mathrm{b} \sin \mathrm{w} \mathrm{t}$.
8. a) Explain Markov Process in details. 7
b) Explain wide sense Stationary Process.

