M.Sc.-II (Mathematics) (C.B.C.S. Pattern) Sem-IV

PSCMTHT20.3 - Coding Theory Paper-XX (Optional)

P. Pages : 2 **GUG/S/19/11405**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that : for a BSC with crossover probability p < ½, the maximum likelihood decoding rule is the same as the nearest neighbor decoding rule. **10**

b) Suppose that codewords from the binary code {000, 100, 111} are being sent over BSC with crossover probability P = 0.03. Use the maximum likelihood decoding rule to decode the following received words : **10**
i) 010 ii) 011

OR

c) Define. **10**
i) Distance of a code. ii) (n,m,d) – code.
Find the distance of the ternary code
C = {00000, 00111, 11111}.

d) Prove that = A code with distance d is an exactly (d-1) – error – detecting code. **10**

UNIT – II

2. a) If C is a linear code and H a parity-check matrix for C. Then prove that **10**
i) C has distance $\geq d$ if and only if any d-1 columns of H are linearly independent.
ii) C has distance $\leq d$ if and only if H has d columns that are linearly dependent.

b) If $x, y \in F_2^n$, then prove that $\omega t(x+y) = \omega t(x) + \omega t(y) - 2\omega t(x * y)$. **10**

OR

c) If C is a linear code of length n over F_q, the prove that **10**

i) $|C| = q^{\dim(c)}$

ii) $C^{\perp}$ is a linear code and $\dim(C) + \dim(C^{\perp}) = n.$

iii) $(C^{\perp})^{\perp} = C.$

c) If C is a linear code over F_q. Then prove that $d(c) = \omega t(c)$. **10**

UNIT – III

3. a) If I is a non-zero ideal in $F_q[x]/(x^n - 1)$ and g (x) a nonzero monic polynomial of the least degree in I. Then prove that g (x) is a generator of I and divides $x^n - 1$. **10**

b) Let g (x) be the generator polynomial of an ideal of $F_q[x]/(x^n - 1)$. Then prove that the corresponding cyclic code has dimension k if the degree of g (x) is n-k. **10**

OR

c) Let g (x) be the generator polynomial of a q-ary [n, k] – cyclic code C. Put $h(x) = (x^n - 1)/g(x)$. Then prove that $h_0^{-1} h_R(x)$ is the generator polynomial of $C^{\perp}$, where h_0 is the constant term of h (x). **10**

d) Write down steps of decoding algorithm for cyclic codes. **10**

UNIT – IV

4. a) Prove that a narrow-sense binary BCH code of length $n = 2^m - 1$ and designed distance $\delta = 2t + 1$ has dimension atleast $n - m(\delta - 1)/2$. **10**

b) Prove that a BCH code with designed distance δ has minimum distance atleast δ. **10**

OR

c) Prove that a nonzero element r of F_p is a nonzero quadratic residue modulo p if and only if $r \equiv a^2 \pmod{p}$ for some $a \in F_p^*$. **10**

d) Prove that two polynomials $g_Q(x)$ and $g_N(x)$ belong to $f_\ell[x]$. **10**

5. a) Define. **5**

i) Communication channel. ii) Memoryless communication channel.

b) Find all possible generator matrices for binary linear code C = {000, 001, 100, 101}. **5**

c) Let C be the binary [7,4] cyclic code generated by $g(x) = 1 + x^2 + x^3$. Find Parity-Check matrix of C. **5**

d) For a finite field F_{11} compute $Q_{11}, N_{11}, 4Q_{11}$ and $2Q_{11}$. **5**

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