# M.Sc.(Mathematics) (CBCS Pattern) Third Semester <br> PSCMTHT14-3 - Paper-XIV (Optional) : Graph Theory 

P. Pages : 2

GUG/W/18/11284
Time: Three Hours


Max. Marks : 100

Notes: 1. Solve all five questions.
2. All questions carry equal marks.

## UNIT - I

1. a) Prove that if $G$ be a non-empty graph with at least two vertices. Then $G$ is bipartite if and only if it has no odd cycles.
b) Prove that in a graph $G$ there is an even number of odd vertices.

## OR

c) Prove that for a tree T with n -vertices then it has precisely n -1 edges.
d) Prove that A graph G is connected if and only if it has a spanning tree.

## UNIT - II

2. a) In Dijkstra's algorithm, if at some stage $\lambda(\mathrm{v})$ is finite for the vertex V then. Prove that there is a path from s to v whose length is $\lambda(\mathrm{v})$.
b) Let G be a graph with n vertices, where $\mathrm{n} \geq 2$. then prove that G has at least two vertices which are not cut vertices.

## OR

c) Prove that: A connected graph $G$ is Euler if and only if the degree of every vertex is even.
d) Prove that Fleury's algorithm produces an Euler tour in an Euler graph G.

## UNIT - III

3. a) Let G be a plane graph with n vertices, e edges, f faces and k connected components, then prove that $\mathrm{n}-\mathrm{e}+\mathrm{f}=\mathrm{k}+1$.
b) State and prove Euler's formula.

## OR

c) Show that if a planar graph $G$ of order $n$ and size $m$ has $r$ regions and $k$ components, then $\mathrm{n}-\mathrm{m}+\mathrm{r}=\mathrm{k}+1$.
d) Let $G$ be a connected plane graph with $n$ vertices, e edges and $f$ faces. Let $n^{*}$, $e^{*}$ and $f^{*}$ denote the number of vertices, edges and faces respectively of $\mathrm{G}^{*}$ then prove that $n^{*}=f, e^{*}=e$, and $f^{*}=n$

## UNIT - IV

4. a) Prove that if D be a weakly connected digraph with at least two vertices. Then D has a directed Euler trail if and only if $D$ has two vertices $u$ and $v$ such that $\operatorname{od}(\mathrm{u})=\mathrm{id}(\mathrm{u})+1$ and $\mathrm{id}(\mathrm{v})=\operatorname{od}(\mathrm{v})+1$
and, for all other vertices W of $\mathrm{D}, \mathrm{od}(\mathrm{w})=\mathrm{id}(\mathrm{w})$, furthermore, in this case the trail begins at $u$ and ends at $v$.
b) Prove that : every tournament T has a directed Hamiltonian path.

## OR

c) Find the orientation of the graph.

d) Let $u$ and $v$ be two distinct vertices of the graph G. Prove that
i) A set $S$ of vertices of $G$ is $u$-v separating if and only if every $u$-v path has at least one internal vertex belonging to $S$.
ii) A set F of edges of G is $\mathrm{u}-\mathrm{v}$ separating if and only if every u -v path has at least one edge belonging to F .
5. a) Let G be a graph with n vertices and e edges and let m be the smallest positive integer such that $m \mathrm{Z} \frac{2 \mathrm{e}}{\mathrm{n}}$-Prove that G has a Vertex of degree at least m .
b) Write down the steps involved in Dijkstra's algorithm.
c) Let $G_{1}$ and $G_{2}$ be two plane graphs which are both redrawing's of the same planar graph G. Then prove that $f\left(\mathrm{G}_{1}\right)=\mathrm{f}\left(\mathrm{G}_{2}\right)$.
d) Prove that a simple graph $G$ is Hamiltonian if and only if its closure $\mathrm{C}(\mathrm{G})$ is Hamiltonian.

