M.Sc.(Mathematics) (C.B.C.S. Pattern) Sem-III

PSCMTHT14-3-Paper-XIV (Optional) : Graph Theory

P. Pages : 2 **GUG/S/19/11284**

Time : Three Hours * 3 2 1 6 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that for any two vertices u and ν of a graph G, every $u-\nu$ walk contains a $u-\nu$ path. **10**

b) Let G be a simple graph with n vertices and let $\bar{G}$ be it's compliment. Then Prove that **10**

i) For each vertex ν in G, $d_G(\nu) + d_{\bar{G}}(\nu) = n-1$

ii) Suppose that G has exactly one even vertex. How many odd vertices does $\bar{G}$ have

OR

c) Prove that Let G be a graph with n vertices. Then the following three statements are equivalent. **10**

i) G is a tree

ii) G is acyclic graph with n–1 edges.

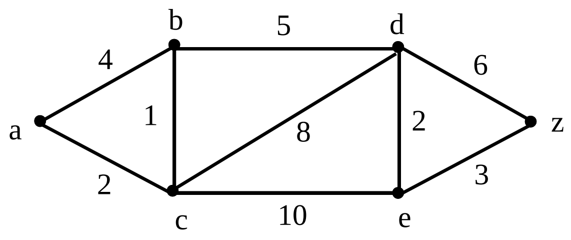
iii) G is a connected graph with n–1 edges.

d) Let G be a acyclic graph with n vertices and K connected components. Then Prove that G has n-K edges. **10**

UNIT – II

2. a) Let G be a weighted connected graph in which the weights of the edges are all non-negative numbers. Let T be a subgraph of G obtained by Kruskal's algorithm. Then prove that T is a minimal spanning tree of graph G. **10**

b) Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the weighted graph. **10**



OR

c) Prove that if G be a graph with n vertices, where $n \ge 2$. Then G has atleast two vertices which are not cut vertices. **10**

d) Prove that: A connected graph G is Euler if and only if the degree of every vertex is even. **10**

UNIT – III

3. a) State and prove Euler's Formula. **10**

b) Let G be a plane graph without loops. If G has a Hamiltonian cycle C and α_i, β_i then prove that $\sum_i (i-2)(\alpha_i - \beta_i) = 0$ **10**

Where, α_i denotes the number of faces of degree i lying inside the cycle C and β_i denotes the number of faces of degree i lying outside the cycle C.

OR

c) Let G be a connected plane graph with n vertices, e edges and f faces. Let n^*, e^* and f^* denotes the number of vertices, edges and faces respectively of G^*, then prove that $n^* = f$, $e^* = e$ and $f^* = n$. **10**

d) Prove that K_y and $K_{z,2}$ are Planar. **10**

UNIT – IV

4. a) Define **10**

i) Directed graph.

ii) Weakly connected digraph.

iii) Strongly connected digraph.

iv) Simple digraph.

v) Euler digraph.

b) Prove that: A tournament T is Hamiltonian if and only if it is strongly connected. **10**

OR

c) State and prove Max – Flow Min cut theorem. **10**

d) Prove that A simple graph G is n-connected if and only if given any pair of distinct vertices u and ν of G, there are at least n internally disjoint paths from u to ν. **10**

5. a) Prove that any tree T with at least two vertices has more than one vertex of degree 1. **5**

b) Write a short note on Chinese Postman problem. **5**

c) Prove that K_5 is nonplanar. **5**

d) Prove that an acyclic digraph has atleast one vertex of out degree zero. **5**

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