M.Sc. (Mathematics)(CBCS Pattern)-Regular-Semester 2017 Sem III

PSCMTHT14-3 : Graph Theory (Optional) Paper-XIV

P. Pages : 2 **GUG/S/18/10106**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that in a graph G there is an even number of odd vertices. **10**

b) Define self-complementary graph. Prove that if G is a self – Complementary graph with n-vertices then n is either 4 t or 4t + 1 for some integer t. **10**

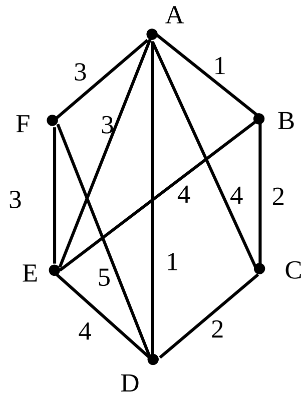
OR

c) Given any two vertices u and v of a graph G, every u-v walk contains a u-v path. **10**

d) If T is a tree with n-vertices then it has precisely n-1 edges. **10**

UNIT – II

2. a) Find minimal spanning tree of the graph by Prim's Algorithm. **10**

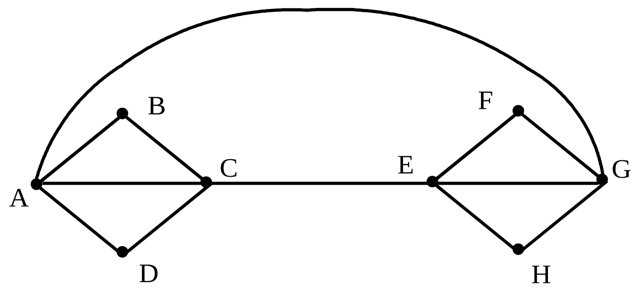


b) Prove that if G is a graph in which the degree of every vertex is at least two then G contains a cycle. **10**

OR

c) A connected graph G is Euler if and only if the degree of every vertex is even. **10**

d) Use Fleury's algorithm to construct an Euler circuit for the graph. **10**



UNIT - III

3. a) Prove that $K_{3,3}$ is non-planar. **10**

b) Let G be a plane graph with n vertices, e edges, f faces and k connected components, then $n-e+f=k+1$. **10**

OR

c) Show that if a planar graph G of order n and size m has r regions and k components, then $n-m+r=k+1$. **10**

d) Prove that k_4 and $K_{2,2}$ are planar. **10**

UNIT – IV

4. a) State and prove first theorem of Digraph theory. **10**

b) Let D be a weakly connected digraph with atleast two vertices. Then D has a directed Euler trail if and only if D has two vertices such that. **10**

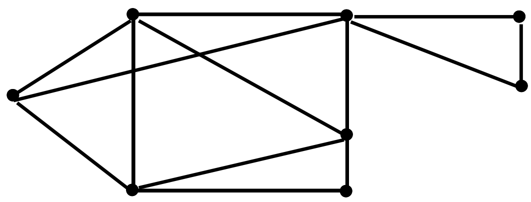
$od(u)=id(u)+1$ and

$id(v)=od(v)+1$

and for all other vertices w of D.

OR

c) Find the orientation of the graph. **10**



d) Define **10**

i) Directed graph.
ii) Simple digraph.
iii) Euler digraph.
iv) Weakly connected digraph
v) Strongly connected diagraph.

5. a) Prove that if an edge e is not part of any cycle in G then e is a bridge. **5**

b) Write a note on Kruskal's Algorithm. **5**

c) Let G_1 and G_2 be two plane graphs which are both redrawings of the same planar graph G. Then $f(G_1)=f(G_2)$. **5**

d) A digraph G is an Eulerian digraph if and only if G is connected and is balanced. **5**

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