M.Sc. - II (Mathematics) (C.B.C.S. and Old) Sem-III

PSCMTHT14-2 / MSc2394-Optional Paper-IV : General Relativity-I

P. Pages : 2 **GUG/S/19/11283**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that any tensor of second order may be expressed as sum of a symmetric & skew symmetric tensor. **10**

b) Show that: **10**

i) $[ij, k] = [ji, k]$

ii) $\left\{ {i \atop jk} \right\} = \left\{ {i \atop kj} \right\}$

iii) $\left\{ {i \atop ij} \right\} = \frac{\partial}{\partial xj} \log \sqrt{g}$

OR

c) Let A^r, B^r be arbitrary contravariant vectors & $A^r B^s$ be an invariant then show that Q_{rs} is a component of covariant tensor of order 2. **10**

d) Prove that the Christoffel symbols of second kind are not tensor. **10**

UNIT – II

2. a) Derive the relation between Newton's gravitation potential $\psi \,\&\, g_{44}$. **10**

b) Derive the energy momentum tensor of a perfect fluid in the form. **10**

$T^i_j = (\rho + p)\, u^i u^j - Pg^{ij}$

OR

c) Derive the relation, **10**

$\nabla^2 \psi = 4\pi G\rho$, where g = 1.

d) Explain the another implication of co-variance with respect to **10**

i) Geometry of special relativity.

ii) The test particle trajectory.

UNIT – III

3. a) Obtain the equation of planetary orbit, $\mu'' + \mu = 3\mu^2 M + \frac{M}{h^2}$ **10**

b) State one of the classic test of general relativity & explain it. **10**

OR

c) Show that every solution of field equation corresponding to the field is static. **10**

d) Obtain the Schwarzschild's solution in isotropic coordinate system. **10**

UNIT – IV

4. a) Obtain the Einstein's field equation **10**

$$R_{ij} = \lambda g_{ij}$$

from the Poisson's equation.

b) Derive the line element of the interior Schwarzschild solution. **10**

OR

c) Derive the gravitational field equation for nonempty space. **10**

d) Derive the linearized field equation. **10**

5. a) Show that **5**

$$A^i_i = \frac{1}{\sqrt{g}} \frac{\partial\left(A^i \sqrt{g}\right)}{\partial x^i}$$

b) Explain the principle of covariance. **5**

c) State the Birkhoff's theorem in two forms. **5**

d) Discuss the associated Weyl's solution. **5**

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