M.Sc. I (Mathematics)(with Credits)-Regular-Semester 2012 (Old / CBCS Pattern) Sem II

0169 / PSCMTHT07 : Real Analysis Paper-II

P. Pages : 2 GUG/S/18/5767

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Let $\{E_i\}$ be a sequence of measurable sets. If the sets $\{E_n\}$ are pairwise disjoint, then show that $m\left(\bigcup_i E_i\right) = \sum_i mE_i$ **10**

b) Prove that the outer measure of an interval is its length. **10**

OR

c) If E_1 and E_2 are measurable then prove that $E_1 \cup E_2$ is measurable. **10**

d) Prove that the interval (a, ∞) is measurable. **10**

UNIT – II

2. a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$ and if $\phi \geq \psi$ a. e. Then $\int \phi \geq \int \psi$. **10**

b) Let f be a bounded function defined on [a , b] If f is Riemann integrable in [a , b] then prove that f is measurable and **10**

$$R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx$$

OR

c) State and prove bounded convergence theorem. **10**

d) Let f be an integrable function on [a ,b] and suppose that $F(x) = F(a) + \int_a^x f(t)\,dt$ then prove that $F'(x) = f(x)$ for almost all x in [a, b] **10**

UNIT – III

3. a) Prove that a function F is an indefinite integral iff it is absolutely continuous. **10**

b) If f is absolutely continuous on [a , b] & $f'(x) = 0$, a. e. then prove that f is constant. **10**

OR

c) State and prove the Holder's inequality. **10**

d) Prove that the ℓ^p spaces are complete. **10**

UNIT – IV

4. a) Prove that, a metric space X is compact if and only if it is both complete and totally bounded. **10**

b) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then show that f is uniformly continuous. **10**

OR

c) Prove that a sequentially compact metric space is totally bounded. **10**

d) Prove that, a 6 – compact locally compact space is paracompact. **10**

5. a) Prove that if $m^* A = 0$ then $m^* (A \cup B) = m^* B$. **5**

b) Let f be a non-negative measurable function, then show that $\int f = 0 \Rightarrow f = 0$ a. e. **5**

c) State & prove Jensen inequality. **5**

d) Prove that the continuous image of a compact set is compact. **5**

****