M.Sc. - I (Mathematics)(with Credits)-Regular-Semester 2012 Sem II (Old)

# 0174 - (Optional) Paper-VI : Mathematical Methods

P. Pages : 2 **GUG/S/18/5766**

Time : Three Hours * 1 7 8 6 * Max. Marks : 100

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Notes : 1. Solve **all five** questions.
2. Each questions carries equal marks.

**UNIT - I**

**1.** a) If $f(x) \in C[a, b]$ where $0 < a < b < \infty$ then prove that $\int_a^b f(x)\sin(\lambda t)\,dt \to 0$ as $\lambda \to \infty$. **10**

b) Find the Fourier sine and cosine transform of $f(t) = \frac{e^{-at}}{t}$. **10**

**OR**

c) Find $F_C\left[e^{-x^2}; \xi\right]$? **10**

d) Prove that $F[f.g; \xi] = F(\xi) \cdot G(\xi)$ where $F(\xi)$ and $G(\xi)$ denote the Fourier transform of f and g respectively. **10**

**UNIT - II**

**2.** a) If $L[f(t), P] = \overline{f}(p)$ and $L[g(t), P] = \overline{g}(p)$ then prove that $L[f * g, p] = \overline{f}(p) \cdot \overline{g}(p)$ **10**

b) State & prove existence theorem for Laplace transform. **10**

**OR**

c) If $L[f(t)] = \overline{F}(p)$ then prove that $L\left[t^n \cdot F(t)\right] = (-1)^n \cdot \frac{d^n}{dp^n} \overline{F}(p),\ n = 1, 2, 3, \ldots.$ **10**

d) Solve by Laplace transform **10**
$y'' - 4y' + 3y = f(x),\quad y(0) = 1,\ y'(0) = 0$

**UNIT - III**

**3.** a) Find the Finite Fourier sine transform of $\cos(ax)$. **10**

b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 4,\ t > 0$ **10**

subject to condition $u(0, t) = u(4, t) = 0\ u(x, 0) = 2x$.

**OR**

c) Express the function **10**

$$f(x) = 1, \text{ for } |x| \le 1$$
$$= 0, \text{ for } |x| > 1$$

as a Fourier integral and hence evaluate.

d) Prove that : **10**

$$F_S[x(a-x), n] = \frac{2a^3}{n^3\pi^3}\left[1+(-1)^{n+1}\right]$$

**UNIT - IV**

**4.** a) Find the Hankel finite transform for $x^2$ if $x J_0(\xi_n \cdot x)$ is the Kernel of the transform. **10**

b) Prove that **10**

$$h_1, \nu\left[x^\nu; n\right] = \frac{a^{v+1}}{\xi_n} \cdot J_{v+1}(\xi_n \cdot a).$$

**OR**

c) Find the Hankel transform of **10**

$$f(x) = a^2 - x^2, \; 0 < x < a, \; n = 0$$
$$= 0 \qquad , \; x > a \quad , \; n = 0$$

d) Define Mellin transform and calculate **10**

i) $M\left[e^{-\alpha x}\right], \alpha > 0$ ii) $M[F(ax)]$

**5.** a) If $\lambda$ is constant, then show that **5**

$$F\left[e^{i\lambda t} \cdot f(t), \xi\right] = F[f(t); \xi + \lambda]$$

b) $L\left[\frac{\sin at}{t}\right]$ **5**

c) Find $F_s[1; n]$. **5**

d) Find the Hankel transform of $\frac{1}{x}$ then apply the inversion formula to get original function. **5**

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