M.SC. - I (Mathematics) Second Semester Old+CBCS

0172 / PSCMTHT09 - Mathematics Paper-IV (Classical Mechanics)

P. Pages : 2 **GUG/W/18/11214**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** question.
2. Each question carries equal marks.

UNIT - I

1. a) Show that Hamilton's principle is a necessary and sufficient condition for Lagrange's equation. **10**

b) By the minimum surface of revolution obtain the equation of catenary. **10**

OR

c) Derive Lagrangian equation from Hamilton principle. **10**

d) Discuss the Brachistochrone problem. **10**

UNIT - II

2. a) If the constraint are independent of time for the equation - **10**

$\bar{r}_i = \bar{r}_i(q_1, q_2, ...q_n, t)$

do not involve time t explicitly then show that $\Delta\int 2T\,dt = 0$

b) Discuss the principle at least action. **10**

OR

c) Discuss the Routh's procedure and show that the nonignorable coordinate obey the Lagrange equation **10**

$$\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_i}\right) - \frac{\partial R}{\partial q_i} = 0 \quad i = 1, 2, ...$$

d) Obtain the canonical equations of Hamilton. **10**

UNIT - III

3. a) Obtain the equation of the canonical transformation. **10**

b) If $f = f_1(q, Q, t)$ and $f = f_2(q, P, t)$ are generating functions of canonical transformation then prove that - **10**

i) $K = H + \frac{\partial f_1}{\partial t}$ and ii) $K = H + \frac{\partial f_2}{\partial t}$

OR

c) Prove that the value of the Poisson bracket [Q, P] implies the sympletic condition. **10**

d) Explain the sympletic approach to canonical transformation and obtain necessary condition, $MJ\tilde{M} = J$. **10**

UNIT - IV

4. a) Show that the density of the system in the neighborhood of some given system in phase space remains constant in time. **10**

i.e. $\frac{\partial D}{\partial t} = 0$ or $\frac{\partial D}{\partial t} = -[D, H]$.

b) Explain the angular momentum Poisson bracket relation. **10**

OR

c) Prove that, the generating function G corresponding to an infinitesimal rotation of the mechanical system about an axis denoted by the unit vector n is given by G = L.n where L is the total angular momentum of the system. **10**

d) In a symmetry group of mechanical system obtain the identities **10**

$\left[L_i, L_j\right] = \in_{ijk} \cdot L_k$

$\left[D_i, L_j\right] = \in_{ijk} \cdot D_k$

$\left[D_i, D_j\right] = \in_{ijk} \cdot L_k$

5. a) Prove that the shortest distance between the two point in a plane is a straight line. **5**

b) If the generalized co-ordinate does not appear in H, then prove that the corresponding momentum is conserved. **5**

c) Show directly that the transformation $Q = \log\left(\frac{1}{q} \sin p\right)$, $p = q \cot p$ is canonical. **5**

d) Prove that the Poisson bracket of constant of the motion is itself a constant of the motion even when the constant depends on time explicitly. **5**

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