

M.Sc. (Physics) (C.B.C.S. Pattern) Sem-I
PSCPHYT01 - Core-I - Paper-I : Mathematical Physics

P. Pages : 2

Time : Three Hours



GUG/S/19/11179

Max. Marks : 80

Notes : 1. All questions are compulsory.

1. Either

a) Prove that $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational. 8

b) Find the Fourier series for 8

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

OR

e) Find sine and cosine transform of 8

$$f(x) = \begin{cases} 1 + x/a & -a < x < 0 \\ 1 - x/a & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

f) The fluid motion is given by $\vec{v}(y \sin z - \sin x)i + (x \sin z + dyz)j + (xy \cos z + y^2)k$ is the motion is irrotational. If so find velocity potential. 8

2. Either

a) If there be an entity represented by multi suffix set a_{ij} relatively to any given system of rectangular axes and if $a_{ij} b_i$ is a vector, where b_i is any arbitrary vector whatsoever then a_{ij} is a tensor of order two. 6

b) i) $\text{grad}(\vec{f} \cdot \vec{g}) = \vec{f} \times \text{curl} \vec{g} + \vec{g} \times \text{curl} \vec{f} + \vec{f} \cdot \Delta \vec{g} + \vec{g} \cdot \Delta \vec{f}$ 10

ii) $\text{curl}(\vec{f} \times \vec{g}) = \vec{f} \text{div} \vec{g} - \vec{g} \cdot \text{div} \vec{f} + \vec{g} \cdot \Delta \vec{f} - \vec{f} \cdot \Delta \vec{g}$

OR

e) If $a_{ijkl} \dots$ is a tensor of order m then set obtained an identifying any two suffixes is a tensor of order $(m-2)$. 6

f) Show that if $a_{ijkl} \dots$ is symmetric (skew symmetric) in any two suffixes, then so is also $\bar{a}_{pqrs} \dots$ in the same suffix. 6

g) What is mixed tensor of second rank. Prove that δ_q^p is a mixed tensor of the second rank. 4

3. Either

a) Find eigen value of A^3 if 6

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

- b) Satisfy the Cayley-Hamilton theorem and find A^{-1} of the matrix 10
- $$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$$

OR

- e) Find the eigen value & eigen vector of matrix 8
- $$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

- f) Diagonalise $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ 8

4. Either

- a) Solve the differential equations. 8

i) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$ ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$

- b) $3x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0$ 8

Find the solution in generalized series form about $x = 0$ of differential equation.

OR

- e) Prove that 4
- i) $x J_n' = n J_n - x J_{n+1}$

ii) $x J_n' = -n J_n + x J_{n-1}$ 4

- f) Find the power series solution of 8
- $$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

5. Answer the following.

- a) Find the Fourier expansion of time period of 2π , where $f(x) = x^2$, $-\pi < x < \pi$. 4

- b) Prove that $\epsilon_{ilm} \epsilon_{jmn} = 2\delta_{ij}$ 4

- c) Find A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$ 4

- d) Solve differential equation $\sin x \frac{dy}{dx} + 2y = \tan^3(x/2)$ 4
