

M.Sc.(I) (Mathematics)(with Credits)-Regular-Semester 2012 Sem I (Old)
0166 - Mathematics : Paper-V : Integral Equations

P. Pages : 3

Time : Three Hours



GUG/S/18/3473

Max. Marks : 100

- Notes : 1. All questions carry equal marks.
 2. Solve all the **five** questions.

UNIT - I

1. a) Find the integral equation formulation for 10

$$\frac{d^2y}{dx^2} + 4y = f(x), \quad 0 \leq x \leq \frac{\pi}{2}$$

with b.c.s. $y=0$ at $x=0$ & $y'=0$ at $x=\frac{\pi}{2}$.

- b) Transform the DE to integral equation, 10

$$f - y'' - \lambda y = \cos x, \quad y=0 \text{ at } x=0 \text{ & } y'=0 \text{ at } x=1$$

OR

- c) Solve the integro D.E. 10

$$u'(x) + \int_0^1 e^{x-y} u(y) dy = f(x), \quad 0 \leq x \leq 1 \text{ & } u(0) = 0$$

- d) Convert the DE 10

$$y''(x) + a_1(x)y'(x) + a_2(x)y(x) = f(x)$$

where $y(0) = y_0$ & $y'(0) = y_1$ into an integral equation.

UNIT - II

2. a) Obtain the solution of Fredholm integral equation of second kind whose kernel is Green's function type. 10

- b) Solve the integral equation 10

$$3\sin x + 2\cos x = \int_{-\pi}^{\pi} \sin(x+y)\phi(y)dy.$$

OR

- c) Find the eigen values & eigen functions of 10

$$\phi(x) = \lambda \int_0^1 (1+xt)\phi(t)dt, \quad 0 \leq x \leq 1$$

d) Solve the system of equations :

10

$$\phi_1(x) = 1 + \int_0^x \phi_2(y) dy \quad \& \quad \phi_2(x) = e^{2x} - \int_0^x e^{2(x-y)} \phi_1(y) dy$$

UNIT - III

3.

a)

10

$$\text{Solve : } \int_0^x \phi(x-y)[\phi(y) - 2 \sin ay] dy = x \cos ax .$$

b)

10

$$\text{Solve : } \phi(x) = 2 \cos ax + \int_0^x (x-t) \phi(t) dt, \quad x \geq 0 .$$

OR

c)

Find the Fourier series for the equation

10

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1-\alpha^2}{1-2\alpha \cos(x-y)+\alpha^2} \phi(y) dy$$

$$0 \leq \alpha \leq 1, \quad -\pi \leq x \leq \pi$$

d)

Solve the integral equation,

10

$$\phi(x) = 3 \int_0^x \cos(x-y) \phi(y) dy + e^x .$$

UNIT - IV

4.

a)

Obtain the Hilbert transform of a rectangular pulse

10

$$f(t) = 1 \text{ for } |t| < a$$

$$= 0 \text{ for } |t| > a .$$

b)

Find a two term approximation to the solution of integral equation

10

$$y(x) + \int_0^1 k(x,t) y(t) dt = 1$$

$$k(x,t) = x, \quad x \leq t$$

$$= t, \quad x \geq t$$

by using Galerkin's method.

OR

10

c) Solve: $\frac{1}{a^2 + x^2} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(u)}{x-u} du, \quad a > 0.$

10

- d) Calculate an approximation of the form $a_0 + a_1 y$ to $\phi(y)$ is given by

$$\int_0^1 e^{xy} \phi(y) dy = (1+x)^{-1} [e^{x+1} - 1], \quad 0 \leq x \leq 1$$

5. a) Show that $\int_0^\pi \sin(x+y)\phi(y) dy = e^{2x}$, $0 \leq x \leq \pi$ is not self-consistent & so does not have

5

a solution.

- b) Show that if the eigen values exists they are real.

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- c) Solve the integral equation,

$$\phi(x) = \lambda \int_0^x [1 + \phi(y)^2] dy$$

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- d) Solve the integral equation,

$$\int_0^\ell \frac{h(u)}{u-\omega} du = 1, \quad 0 \leq \omega \leq \ell$$

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