M.Sc. (Mathematics) Sem-I (Old)

0167-Optional Paper-VII
Fuzzy Mathematics-I

P. Pages : 3 **GUG/S/19/2185**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Let A, B $\in \mathcal{F}(X)$. Then for all $\alpha \in [0, 1]$ prove that **10**

i) $A \subseteq B$ iff ${}^{\alpha}A \subseteq {}^{\alpha}B$

ii) $A \subseteq B$ iff ${}^{\alpha+}A \subseteq {}^{\alpha+}B$

b) Let $f : X \to Y$ be an arbitrary crisp function. Then prove that for any A $\in \mathcal{F}(X)$ and all $\alpha \in [0, 1]$ the following properties of f fuzzified by the extension principle hold. **10**

i) ${}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$ ii) ${}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$

OR

c) Prove that every fuzzy complement has at most one equilibrium. **10**

d) Let C be a function from [0, 1] to [0, 1]. Then prove that C is a fuzzy complement iff there exists a continuous function f from [0, 1] to R such that f(1) = 0, f is strictly decreasing and $c(a) = f^{-1}(f(0) - f(a))$ for all $a \in [0, 1]$. **10**

UNIT - II

2. a) Let A $\in \mathcal{F}(R)$. Then prove that A is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \phi$ such that **10**

$$A(x) = \begin{cases} 1 & \text{for} \quad x \in [a, b] \\ \ell(x) & \text{for} \quad x \in (-\infty, a) \\ r(x) & \text{for} \quad x \in (b, \infty) \end{cases}$$

where ℓ is a function from $(-\infty, a)$ to [0, 1] that is monotonic increasing, continuous from the right and such that $\ell(x) = 0$ for $x \in (-\infty, w_1)$, r is a function from (b, ∞) to [0, 1] that is monotonic decreasing, continuous from the left and such that r(x) = 0 for $x \in (w_2, \infty)$.

b) Let MIN and MAX be binary operations on R defined by **10**

$$\text{MIN}(A, B)(z) = \sup_{z = \min(x, y)} \min[A(x), B(y)]$$ and

$$\text{MAX}(A, B)(z) = \sup_{z = \max(x, y)} \min[A(x), B(y)]$$ for all $z \in R$ respectively. Then prove that for any $A, B \in R$ MIN [A, MAX(A, B)] = A.

OR

c) Let MIN and MAX be binary operations on R defined by **10**

$\mathrm{MIN}(A, B)(z) = \sup\limits_{z=\min(x, y)} \min[A(x), B(y)]$ and

$\mathrm{MAX}(A, B)(z) = \sup\limits_{z=\max(x, y)} \min[A(x), B(y)]$ for all $z \in R$ respectively. Then prove that for any $A, B, C \in R$

MIN [A, MAX (B, C)] = MAX [MIN (A, B) , MIN (A, C)]

d) Find the solution for the equation $A \cdot X = B$ where A and B are triangular shape fuzzy numbers given by - **10**

$$A(x) = \begin{cases} 0 & \text{for} \quad x \le 3 \text{ and } x > 5 \\ x-3 & \text{for} \quad 3 < x \le 4 \\ 5-x & \text{for} \quad 4 < x \le 5 \end{cases}$$

$$B(x) = \begin{cases} 0 & \text{for} \quad x \le 12 \text{ and } x > 32 \\ (x-12)/8 & \text{for} \quad 12 < x \le 20 \\ (32-x)/12 & \text{for} \quad 20 < x \le 32 \end{cases}$$

UNIT - III

3. a) Write four fundamental concepts associated with fuzzy partial orderings relations and further write the properties of every partial ordering relations. **10**

b) Prove that for any fuzzy relation R on X^2, the fuzzy relation $R_{T(i)} = \bigcup\limits_{n=1}^{\infty} R^{(n)}$ is the i-transitive closure of R. **10**

OR

c) Let R be a reflexive fuzzy relation on X^2, where $|X| = n \ge 2$ Then prove that $R_{T(i)} = R^{(n-1)}$. **10**

d) Prove that for any $a, a_j, b, d \in [0, 1]$, where j takes values from an index set J operation w_i has the following properties. **10**

i) $i(a, b) \le d$ iff $w_i(a, d) \ge b$

ii) $w_i[i(a, b), d] = w_i[a, w_i(b, d)]$

iii) $w_i\left[\sup\limits_{j\in J} a_j, b\right] = \inf\limits_{j\in J} w_i(a_j, b)$

UNIT - IV

4. a) Prove that it $S(Q, R) \ne \phi$ for $P \overset{i}{o} Q = R$ then $\hat{P} = (Q \overset{w_i}{o} R^{-1})^{-1}$ is the greatest member of S(Q, R). **10**

b) Let the t-norm i is employed in $P \overset{i}{o} Q = R$ be the product and let $Q = \begin{bmatrix} .1 \\ .2 \\ .3 \end{bmatrix}$ and $R = \begin{bmatrix} .12 \\ .18 \\ .27 \end{bmatrix}$ **10**

Then find the greatest solution $\hat{P}$.

OR

c) Prove that the fuzzy relation $\tilde{P} = (Q \overset{w_i}{o} R^{-1})^{-1}$ is the best approximation in terms of the goodness index G defined by $G(P') = \left\| P' \overset{i}{o} Q = R \right\|$ of fuzzy relation equations $P \overset{i}{o} Q = R$ **10**

d) Let δ be the solvability index of $P \overset{i}{o} Q = R$ defined by $\delta = \sup\limits_{P \in \mathcal{F}(X \times Y)} \left\{ \left\| P \overset{i}{o} Q = R \right\| \right\}$ **10**

then prove that $\delta \le \operatorname*{int}\limits_{z \in Z} w_i \left(\sup\limits_{x \in X} R(x, z), \sup\limits_{y \in Y} Q(y, z) \right)$

5. a) Prove that the standard fuzzy intersection is the only idempotent t- norm. **5**

b) Explain fuzzy number. **5**

c) Write some fundamental concepts associated with fuzzy partial orderings relations. **5**

d) Explain approximate solutions of fuzzy relations. **5**

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