



- Notes : 1. Solve **all five** questions.  
2. All questions carry equal marks.

**UNIT – I**

1. a) Show that any Covariant tensor of the second order may be expressed as the sum of a symmetric tensor and a skew symmetric tensor. **6**
- b) Let  $A^m$ ,  $B_{mrs}$  are tensors, show that  $A^m B_{mrs}$  is also a tensor. **6**

**OR**

- c) Let  $A^r, B^r$  be arbitrary contravariant vectors and  $a_{rs} A^r B^s$  be an invariant. Then show that  $a_{rs}$  are the components of a Covariant tensor of order two. **6**
- d) Define orthogonal vectors and show that the vectors  $A^m = (1,1,1)$  and  $B^m = (2,-1,-1)$  are orthogonal in the space  $ds^2 = dx^2 + dy^2 + dt^2$  but not in  $ds^2 = dx^2 + dy^2 - dt^2$ . **6**

**UNIT – II**

2. a) Prove that  $g^{mn}_{,r} = -g^{ms}\Gamma_{sr}^n - g^{sn}\Gamma_{sr}^m$  **6**
- b) Show that Covariant derivatives of  $g^{mn}$ ,  $g_{ms}$  and  $\delta_n^m$  vanish. **6**

**OR**

- c) Show that  $R_{prmn;t} + R_{prnt;m} + R_{prtm;n} = 0$ . **6**
- d) Show that the divergence of Einstein's tensor vanishes i.e.  $G^m_{n;m} = 0$ . **6**

**UNIT – III**

3. a) Prove that  $E = mc^2$ . **6**
- b) Show that four velocities and four acceleration are mutually orthogonal. **6**

**OR**

- c) Show that the quantity  $p^2 - \frac{E^2}{C^2}$  is an invariant and whose numerical value is  $-m_0^2 c^2$ . 6
- d) A body of mass  $m$  disintegrates into two parts  $m_1$  and  $m_2$ , while at rest. Show that the energies  $E_1$  and  $E_2$  of the two parts satisfy the relation  $\frac{E_1}{E_2} = \frac{m^2 + m_1^2 - m_2^2}{m^2 - m_1^2 + m_2^2}$ . 6

#### UNIT – IV

4. a) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is 6

$$H = \left\{ m_0^2 C^4 + C^2 \left[ p - \left( \frac{e}{c} \right) A \right]^2 \right\}^{1/2} + e \phi$$

- b) Write the Maxwell's equations of electromagnetic theory in vacuum and also express these equations in component form. 6

#### OR

- c) Show that  $T^{ij}$  is symmetric tensor and prove that the energy-momentum tensor of the electromagnetic field is trace free. 6
- d) Find components of  $T^{44}$  in terms of electric & magnetic field strength. 6

5. Solve **any six**.

- a) Show that  $a_{mn} x^m x^n = 0$  for a skew symmetric tensor  $a_{mn}$ . 2
- b) Show that  $\delta_s^r$  is a mixed tensor of order two. 2
- c) Show that  $[mr, n] + [nr, m] = g_{mn,r}$ . 2
- d) Show that  $R_{rm}$  is a symmetric tensor. 2
- e) Show that the four velocity of a particle is a unit time like vector. 2
- f) Show that square of the magnitude of the four momentum vector  $p^i = m_0^2 c^2$ . 2
- g) Write the electromagnetic field tensor  $F_{ij}$ . 2
- h) Write the wave equation for the propagation of the electric field strength  $E$  in free space with velocity  $c$ . 2

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