

Bachelor of Science (B.Sc.)-III Sixth Semester
B.Sc. 4531 / MAT 305 - Mathematics-II
(Optional) (Special Relativity-II) Paper - II

P. Pages : 2

Time : Three Hours



GUG/W/18/1354

Max. Marks : 60

- Notes :
1. Solve all the **five** questions.
 2. Question 1 to 4 has an alternative. Solve each question in full or its alternative in full.
 3. All questions carry equal marks.

UNIT – I

1. a) Define contravariant tensor of order one. Show that gradient of a scalar is a covariant vector. 6
- b) A covariant vector has components $2x - z, x^2y, yz$ in rectangular co-ordinates. Find its covariant components in cylindrical co-ordinates. 6

OR

- c) Show that, if A_{rs} and B_m^{pq} are tensors, $A_{rs} B_m^{pq}$ is a tensor. 6
- d) Let A^r be an arbitrary tensor defined by its indices. If the inner product $A^r B_r$ is an invariant, then prove that B_r is a covariant tensor of order one. 6

UNIT – II

2. a) Prove that $g_{,r}^{mn} = -g^{ms} \left|_{sr}^{-n} - g^{sn} \left|_{sr}^{-m} \right.$ 6
- b) Prove that the covariant derivatives of g_{mn}, g^{mn} and δ_n^m vanish. 6

OR

- c) Define geodesics. Show that the geodesics in three dimensional Euclidean space are straight lines. 6
- d) If $R_{,rmn}^P$ and R_{prmn} are curvature tensors there prove that 6
- i) $R_{,rmn}^P = -R_{,rnm}^P$
 - ii) $R_{,rmn}^P + R_{,mnr}^P + R_{,nrm}^P = 0$
 - iii) $R_{prmn} = -R_{rpnm}$

UNIT – III

3. a) Obtain the mass energy equivalence relation $E = mc^2$. 6
- b) A particle is given a kinetic energy equal to n times its rest energy m_0c^2 . What are its speed and momentum. 6

OR

- c) A body of mass m disintegrates into parts m_1 and m_2 , while at rest. Show that the energies E_1 and E_2 of the two parts satisfy the relation 6

$$\frac{E_1}{E_2} = \frac{m^2 + m_1^2 - m_2^2}{m^2 - m_1^2 + m_2^2}$$

- d) Prove that the four force can be expressed as 6

$$f^i = \left(\frac{\bar{F}}{c\sqrt{1-u^2/c^2}}, \frac{\bar{f} \cdot \bar{u}}{c^2\sqrt{1-u^2/c^2}} \right)$$

UNIT – IV

4. a) Explain the term 'four potential'. Obtain transformation of the electromagnetic four potential. 6

- b) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is 6

$$H = \left\{ m_0^2 c^4 + c^2 \left(P - \frac{e}{c} A \right)^2 \right\}^{1/2} + e \phi$$

OR

- c) Obtain transformation for electric field strength F_{14}^1 . 6

- d) Prove that energy momentum tensor of electromagnetic field is trace free. 6

5. Solve **any six**.

- a) Define Mixed tensor of order two. 2
- b) Prove that $\delta_s^r A^s = A^r$. 2
- c) Define Christoffel symbols of the first kind. 2
- d) Define Ricci tensor and Einstein tensor. 2
- e) Define four velocity and four acceleration. 2
- f) Prove that four velocity of a particle is a unit time like vector. 2
- g) Define electromagnetic field tensor. 2
- h) State the Maxwell's equations of electromagnetic theory in vacuum. 2
