B.Sc. III Sem-VI

B.Sc.4530-MAT304-Mathematics Paper-I : Analysis (Compulsory)

P. Pages : 2 **GUG/S/19/1353**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Show that $d(x, y) = |x - y|. \forall x, y \in R$ defines a metric on R. **6**

b) Define a Cauchy sequence and prove that every convergent sequence in a metric space is Cauchy sequence. **6**

OR

c) Let Y be a subspace of a complete metric space X. Then show that Y is complete iff Y is closed. **6**

d) Show that closed subsets of compact sets are compact. **6**

UNIT – II

2. a) Let f, g be bounded functions defined on [a, b] and P be any partition on [a, b]. Then prove that. **6**

$U(P, f+g) \le U(P, f) + U(P, g)$ &

$L(P, f+g) \ge L(P, f) + L(P, g)$

b) Let the function of be defined as **6**

$$f(x) = \begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational} \end{cases}$$

Show that f is not R – integrable over [0, 1] but $|f| \in R[0,1]$.

OR

c) If $f, g \in R[a, b]$ & $f(x) \le g(x) \forall x \in [a, b]$ then prove that **6**

$$\int_a^b f(x)dx \le \int_a^b g(x)dx$$

d) Show that if f be a bounded and integrable function defined on [a, b] with m, M as infimum, supremum respectively then there exists a number μ between m and M such that **6**

$$\int_a^b f(x)dx = \mu(b-a)$$

UNIT – III

3. a) Evaluate $\int_C (z - z^2) dz$, where C is the upper half of the circle |z| = 1. **6**

b) If a function f(z) is analytic in a simply connected domain D, then prove that $\int_C f(z)dz = 0$ for every simple closed curve C in D. **6**

OR

c) Evaluate $\int_c \frac{15z+9}{z(z^2-9)} dz$, where C is the circle |z – 1| = 3 **6**

d) Find the residues at each poles of the function **6**

$$f(z) = \frac{z^2}{(z-1)^2 (z+2)}$$

UNIT – IV

4. a) Find the finite Fourier sine and cosine transforms of mx, $0 < x < \ell$. **6**

b) Prove that **6**

$$\int_0^{\ell} f^1(x) \sin \frac{n\pi x}{\ell} dx = -\frac{n\pi}{\ell} F_c(n)$$

OR

c) Find Fourier sine transform of $f(x) = e^{-|x|}$ $x \ge 0$. Hence show that **6**

$$\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m} m > 0$$

d) Find $F\left[e^{-|x|}\right]$ and hence show that $F\left[e^{-|2x|}\right] = \frac{4}{4+\lambda^2}$. **6**

5. Solve **any six**.

a) Show that $B = (1, \infty)$ is not compact. **2**

b) Define open set. **2**

c) Prove that $m(b-a) \le L(P,f) \le U(P,f) \le M(b-a)$ where M and m denote the sup and inf of f (x) in I = [a, b]. **2**

d) For any partition P, prove that $L(P,f) \le U(P,f)$ **2**

e) Show that $\int_C \frac{1}{z} dz = 2\pi i$, where C is the circle with centre at the origin & radius r. **2**

f) Let C be the curve with the end points a and b show that $\int_C |dz| =$ length of the curve between a and b. **2**

g) Write Dirichlet conditions. **2**

h) Find Fourier sine transforms of $f(x) = K$ where $0 < x < l, K \in N$. **2**

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