



- Notes :
1. Solve all the **five** questions.
 2. Question 1 to 4 has an alternative solve each question in full or its alternative in full.
 3. All questions carry equal marks.

UNIT – I

1. a) Show that in a group G , the mapping $T : G \rightarrow G$, given by $T(x) = x^{-1} \forall x \in G$ is an automorphism of G if and only if G is abelian. If $x_0 \in G$ with $x_0 \neq x_0^{-1}$ then $T \neq I$. **6**
- b) Let G be a group for $g \in G$ define $T_g : G \rightarrow G$ by **6**
 $T_g x = g^{-1} x g, \forall x \in G$
 prove that T_g is an automorphism of G .

OR

- c) Let G be a group and $g \in G$. Functions $T_g : G \rightarrow G, t_g : G \rightarrow G$ are defined by **6**
 $T_g(x) = xg, t_g(x) = gx \forall x \in G,$
 Prove that
 i) $T_g t_h = t_h T_g \forall g, h \in G$
 ii) If $\theta : G \rightarrow G$ is one – one onto such that
 $\theta.T_g = T_g.\theta \forall g \in G$ then $\theta = t_h$ for some $h \in G$.
- d) Prove that, if H, K be subgroups of group G , then HK is a subgroup of G iff $HK = KH$. **6**

UNIT – II

2. a) If in a ring $R, x^3 = x, \forall x \in R$ then show that R is commutative. **6**
- b) If R is a ring with zero element 0 , then prove that for all $a, b, c \in R$. **6**
 i) $a0 = 0a = 0$
 ii) $a(-b) = (-a)b = -(ab)$
 iii) $(-a)(-b) = ab$

OR

- c) Let the characteristic of the ring R be 2 and let $ab = ba \forall a, b \in R$. **6**
 Then show that,
 $(a + b)^2 = a^2 + b^2 = (a - b)^2$

d) Let M be the ring of matrices of order 2 over the field \mathbb{R} of real numbers and

6

$$U = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

$$V = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

$$W = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$$

be non empty subsets of M .

Show that,

- i) U is a right ideal of M but U is not a left ideal.
- ii) V is a left ideal of M but not a right ideal.

UNIT – III

3. a) Show that $P_n(1) = 1$ and $P_n(-x) = (-1)^n P_n(x)$ 6

Hence or otherwise deduce that $P_n(-1) = (-1)^n$.

b) Prove that 6

$$\int_{-1}^1 (x^2 - 1) P_n' P_{n+1} dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

OR

c) Show that 6

$$J_{1/2}'(x) J_{-1/2}(x) - J_{-1/2}'(x) J_{1/2}(x) = \frac{2}{\pi x}$$

d) Prove that 6

$$\int_0^x x^{-n} J_{n+1}(x) dx = \frac{1}{2^n n!} - x^{-n} J_n(x)$$

UNIT – IV

4. a) Find the Fourier Series for the function 6

$$f(x) = e^{ax}, \quad -L < x < L$$

b) Expand the following function in Fourier Series 6

$$f(x) = |x|, \quad x \in (-\pi, \pi)$$

OR

- c) If $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$ 6

Prove that

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$$

- d) Find the Fourier Sine series 6

$$\begin{aligned} f(x) &= \frac{1}{4} - x, \quad 0 < x < \frac{1}{2} \\ &= x - \frac{3}{4}, \quad \frac{1}{2} < x < 1 \end{aligned}$$

5. Solve any six.

- a) Show that for an abelian group G , $I(G) \neq \{I\}$ 2
- b) For $G = S_3$ prove that $G \approx I(G)$. 2
- c) Define the ring. 2
- d) Let R be a ring. Prove that if $a, b \in R$, then 2
 $(a + b)^2 = a^2 + ab + ba + b^2$
 where by x^2 we mean xx .
- e) Show that 2
 $\int_{-1}^1 x^3 P_3(x) dx = \frac{4}{35}$
- f) Show that 2
 $n P_{n-1} = (2n + 1)x P_n - (n + 1)P_{n+1}$
- g) Let f & g be respectively even and odd functions defined on the real line if $f + g = 0$ on R then 2
 Prove that $f(x) = 0 = g(x)$ on R .
- h) Define the periodic function. 2
