B.Sc. II (CBCS Pattern) Sem-III

USMT-05 - Mathematics - I : Real Analysis

P. Pages : 2 **GUG/W/22/11612**

Time : Three Hours * 3 7 5 4 * Max. Marks : 60

Notes : 1. Solve all the **five** question.
2. All questions carry equal marks.

UNIT - I

1. a) Let $X = \langle x_n \rangle$ and $Y = \langle y_n \rangle$ be a sequences of real numbers that converges to x and y respectively then prove that the sequence X + Y converges to x + y. **6**

b) Evaluate $\lim\limits_{n\to\infty} \frac{4^{n+2}+3^{n+1}}{3^{n+2}+4^{n+1}}$ **6**

OR

c) If $\langle s_n \rangle, \langle t_n \rangle$ and $\langle u_n \rangle$ be three sequences. **6**

Such that

i) $s_n \le t_n \le u_n \ \forall_n$

ii) $\lim s_n = \lim u_n = \ell$

then prove that $\lim t_n = \ell$

d) Show that $\lim\limits_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + --- + \frac{1}{\sqrt{n^2+n}}\right] = 1$ **6**

UNIT – II

2. a) Prove that geometric series $\sum\limits_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for 0 < x < 1 and diverges for $x \ge 1$. **6**

b) Test the convergence of the series. **6**

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ------$

OR

c) Test the convergence of series by D'Alembert's ratio test $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + ----$ **6**

d) Test the convergence of the alternating series **6**

$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + -----$

UNIT – III

3. a) Show that the function $d : R \times R \to R$ designed by $d(x, y) = |x^2 - y^2| \forall x, y \in R$ is a pseudo metric on R and not a metric on R **6**

b) Prove that a set A is open if and only if its complement is closed. **6**

OR

c) Suppose that $Y \subseteq X$ then prove that a subset A of Y is open relative to Y if and only if $A = Y \cap G$ for some open subset G of X. **6**

d) Prove that finite union of closed sets is closed. Give counter example to show that arbitrary union of closed sets need not be closed. **6**

UNIT - IV

4. a) Prove that $m(b-a) \le L(p, f) \le U(p, f) \le M(b-a)$ where M and m denotes respective lub and glb of $f(x)$ in $I = [a, b]$. **6**

b) Let f be a function defined on [a, b] such that $|f(x)| \le M \, \forall x \in [a, b]$ where M is a positive number prove that $\int_a^{-b} f(x) dx - \int_{-a}^{b} f(x) dx \le 2M(b-a)$. **6**

OR

c) Let f be continuous and non – negative on [a, b]. Then prove that $F(x) = \int_a^x f(t) dt$ is monotonic increasing in [a, b]. Further more, $\int_a^b f(t) dt = F(b) \ge 0$ and equality holds true only for f is identically zero on [a, b]. **6**

d) Prove that inequality. **6**

$$\frac{2}{17} < \int_{-1}^{2} \frac{x}{1+x^4} dx < \frac{1}{2}$$

5. Attempt **any six**.

a) State Sandwich theorem on sequences. **2**

b) Evaluate $\lim_{n \to \infty} n^{1/n}$. **2**

c) Test the convergence of the series $\sum_{n=1}^{\infty} x_n$ if $x_n = \cos \frac{\pi}{n}$ **2**

d) State necessary condition for convergence of series. **2**

e) Define limit point of the set. **2**

f) Define open set **2**

g) Define Darboux's lower sum of f corresponding to partitions p. **2**

h) Let f be bounded function defined on [a, b] and p be any partition an [a, b]. Prove that $L(p, -f) = -U(p, f)$ **2**

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