



- Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) Prove that if $\lim S_n$ exists, it must be unique. 6
- b) Find the limit of sequence $\langle S_n \rangle$, where $\langle S_n \rangle = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(n+n)^2}$ 6

OR

- c) Prove that every convergent sequence of real numbers is a Cauchy sequence. 6
- d) Show that 6
- $$\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

UNIT - II

2. a) Prove that the series $\sum x_n$ converges if and only if for every $\epsilon > 0 \exists$ a number $M(\epsilon) \in \mathbb{N}$ such that 6
- $$m \geq n \geq M \Rightarrow |x_{n+1} + x_{n+2} + \dots + x_m| < \epsilon$$
- b) Prove that a geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for $0 < x < 1$ and diverges for $x \geq 1$. 6

OR

- c) Let $x_n \geq 0, y_n \geq 0 \forall n \in \mathbb{N}$ and let $\exists m \in \mathbb{N}$ such that 6
- $$x_n \leq k y_n \quad \forall n \geq m, k > 0,$$
- then show that
- i) $\sum y_n$ converges $\Rightarrow \sum x_n$ converges
- ii) $\sum x_n$ diverges $\Rightarrow \sum y_n$ diverges
- d) Define Alternating series and test the convergence of series 6
- $$\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \dots$$

UNIT - III

3. a) Show that $d(x, y) = |x - y| \forall x, y \in \mathbb{R}$ defines a metric on \mathbb{R} . 6
- b) Show that every neighbourhood is an open set. 6

OR

- c) Let $\{A_\alpha\}$ be a finite or infinite collection of sets A_α . Then prove that 6
- $$\left[\bigcup_{\alpha} A_{\alpha} \right]^C = \bigcap_{\alpha} A_{\alpha}^C$$

- d) Prove that every convergent sequence in a metric space is a Cauchy sequence. 6

UNIT – IV

4. a) Show that any constant function defined on a bounded closed interval is integrable. 6

- b) If f is a bounded and integrable over $[a, b]$ and M, m are bounds of f over $[a, b]$, then prove that 6

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$$

OR

- c) Let a function f be continuous and non-negative on the interval $[a, b]$. Prove that 6

$$F(x) = \int_a^x f(t) dt, x \in [a, b] \text{ is monotonic non-decreasing in } [a, b].$$

- d) Prove that if f be a bounded and integrable function defined on $[a, b]$ with m, M as infimum, supremum respectively, then there exists a number μ between m and M such that 6

$$\int_a^b f(x) dx = \mu(b-a)$$

5. Solve any six.

- a) Prove that $\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$ 2

- b) Evaluate $\lim S_n$ for the sequence 2
 $S_n = \sqrt{n+a} - \sqrt{n+b}, a \exists b$

- c) Test the convergence of series by using P-series test 2
 $\sum \frac{1}{\sqrt{n}} = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots$

- d) Test for the convergence of the series 2
 $\sum \left(\frac{n}{n+1} \right)^{n^2}$

- e) Define metric. 2

- f) Define open set. 2

- g) Define Darboux's upper & lower sums. 2

- h) Prove that 2
 $m(b-a) \leq L(p, f) \leq U(p, f) \leq M(b-a)$
