B.Sc. II (With Credits)-Regular-Semester 2012 Sem III

B.Sc.23112 - Mathematics Paper-II (Differential Equations)

P. Pages : 2 **GUG/S/18/3345**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve **all five** question.
2. Question No. 1 to 4 an alternative solve each question in full or its alternative in full.
3. All question carry equal marks.

UNIT - I

1. a) Solve : **6**

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{-y(z^2+x^2)} = \frac{dz}{z(x^2+y^2)}$$

b) Find the general integral of the PDE $z(xp - yq) = y^2 - x^2$. **6**

OR

c) Prove that the PDE $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ where f is homogenous in x, y, z. **6**

d) Solve $Z^2 = pqxy$ by Charpit's method. **6**

UNIT - II

2. a) Solve the partial differential equation **6**

$(D^2 + 3DD' + 2D'^2)z = x + y$.

b) Find the P. I. of **6**

$(D^3 + 4D^2D' - 4DD'^2)z = \cos(2x + 3y)$.

OR

c) Solve $(D^2 + DD' + D' - 1)z = e^{-x} + e^{2x-y}$. **6**

d) Solve the equation **6**

$$x^2 - \frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x.\partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3 \cdot y^4.$$

UNIT - III

3. a) If $c_1, c_2, ...c_n$ are any constant and $f_1(t), f_2(t), ..., f_n(t)$ are functions whose Laplace transforms exits, then show that **6**

$L[c_1f_1(t) + c_2f_2(t), ..., c_nf_n(t) = c_1 L[f_1(t)] + c_2 L[f_2(t)]...... + c_n L[f_n(t)]$.

b) Find $L\left[\sin^2 2t\right]$ and $L\left[\sin^3 at\right]$. **6**

OR

c) Evaluate $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$. **6**

d) Find the inverse Laplace transform of **6**

$$\frac{s^2 + 29}{(s^2 + 4)(s^2 + 9)}.$$

UNIT - IV

4. a) Let the functions $f(t)$ and $g(t)$ satisfy the hypothesis of the existence theorem of their Laplace transforms. If $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(t)$, then prove that $L^{-1}[F(s) \cdot G(s)] = f * g = g * f$. **6**

b) Find the inverse Laplace transform of $\frac{1}{(s^2 + a^2)^2}$ by convolution theorem. **6**

OR

c) Find $L\left(\frac{\sin at}{t}\right)$ and hence show that $\int_0^{\infty} \frac{\sin t}{t} dt = \pi/2$. **6**

d) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3t \cdot e^{-t}$ $y(0) = 4, u'(0) = 2$ by using Laplace transform method. **6**

5. Solve **any six.**

a) Obtain partial differential equation by eliminating the arbitrary constants from the equation $x^2 + y^2 + (z - c)^2 = r^2$. **2**

b) Write the condition of compatibility. **2**

c) Solve the differential equation $yz\,dx + zx\,dy + xy\,dz = 0$. **2**

d) Solve $\left(\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4}\right) = 0$. **2**

e) Define a inverse Laplace transform. **2**

f) Find $L(\sinh at)$ **2**

g) Find $L^{-1}\left[\frac{s+1}{s^2 + 2s + 5}\right]$. **2**

h) Find the Laplace transform of $\frac{e^{-at} - e^{-bt}}{t}$. **2**

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