B.Sc. (I.T.) - I (CBCS Pattern) Sem II

UBITT204 - Paper-IV : Discrete Mathematics

P. Pages : 3 **GUG/S/18/20094**

Time : Three Hours * 2 9 7 4 * Max. Marks : 80

Notes : 1. All questions are compulsory and carry equal marks.
2. Draw neat and labelled diagram and use supporting data wherever necessary.
3. Avoid vague answers and write specific answer related to question.

Either

1. a) i) Let A, B, & C are finite sets, then **8**

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

ii) Let A is a set of A = {a, b, c, d, e} B = {a, b, c, g, h}, C = {b, d, e, g, h, k, m, n} then verify

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |(A \cap B \cap C)|$$

b) If $\begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix}$ find a, b, c, d. **8**

OR

c) i) To show $p \vee (q \wedge r) \equiv (p \wedge q) \wedge (p \vee r)$ **8**

ii) Show that $\sim (p \leftrightarrow q) \equiv ((p \wedge \sim q) \vee (q \wedge \sim p))$

d) Prove that the statement is true by using mathematical induction **8**

$1 + 3 + 5 + \ldots\ldots\ldots\ldots + 2n-1 = n^2$

Either

2. a) Prove that **8**

i) ${}^{n}C_{r} = \frac{n}{r} \times {}^{n-1}C_{r-1}$

ii) $n \times {}^{n-1}C_{r-1} = (n-r+1) \cdot {}^{n}C_{r-1}$

b) Find an explicit formula for the sequence defined by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial conditions $C_1 = 5$ and $C_2 = 3$. **8**

OR

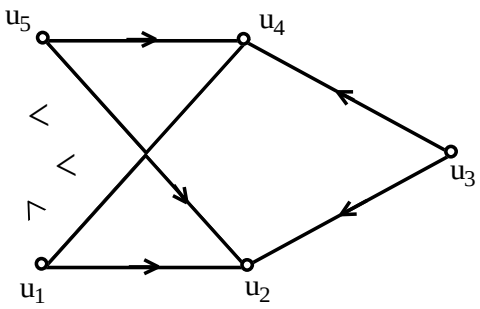
c) Define **8**

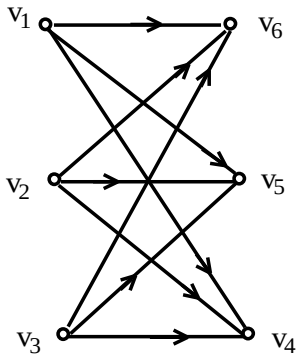
1) One to one function
2) Onto function
3) One to one onto function
4) One one into function
5) Many one into function
6) Equal function

d) Prove that if $f : x \to y$ and $g : y \to z$ be two one to one function, then $g \cdot f$ is also one to one onto function. **8**

Either

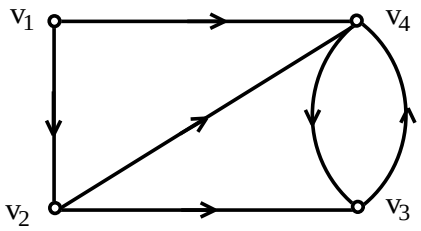
3. a) Show that follow. diagraphs are isomorphic. **8**



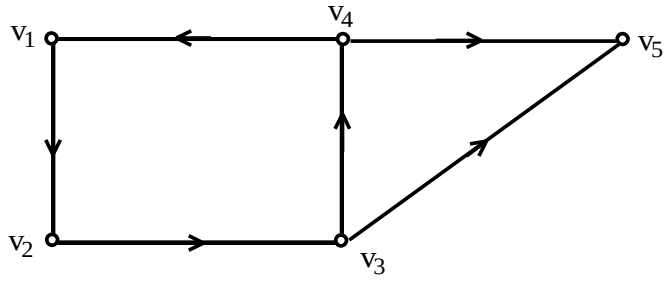


b) Find the node base for a diagraph. **8**

i)



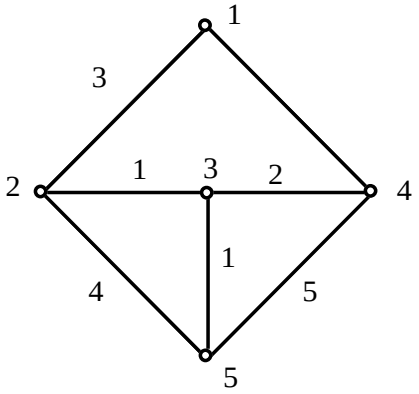
ii)



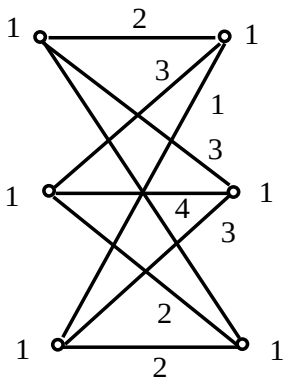
OR

c) Find the minimal spanning tree given by algorithm for each graph. **8**

i)



ii)



d) Let (T, V_0) be a rooted tree then **8**

i) There are two cycles in T.

ii) V_0 is the only root of T.

Either

4. a) Let G is the set of all non zero real numbers and * is a binary operation defined by **8**

$$a * b = \frac{ab}{2}$$

show that $(G, *)$ is an abelian group.

b) Let $(S, *)$ and $(T, *')$ be monoids with identities e and e', respectively. Let $f : S \to T$ be an isomorphism, then $f(e) = e'$. **8**

OR

c) i) Show that $(a^{-1})^{-1} = a$ for all $a \in G$, where G is a group and a^{-1} is an inverse of a. **8**

ii) Show that $(ab^{-1}) = b^{-1} \cdot a^{-1}$ for all $a, b \in G$.

d) i) Show that if G is an abelian group then every subgroup of G is a normal subgroup. **8**

5. Solve all the questions.

a) Show that **4**

i) $A \cap A = A$

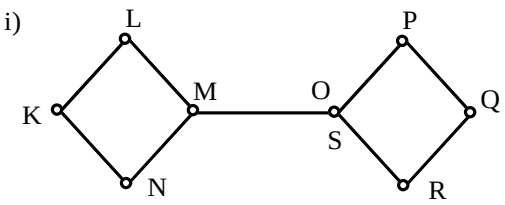
ii) $A \cap \phi = \phi$

iii) $A \cap (B \cap C) = (A \cap B) \cap C$

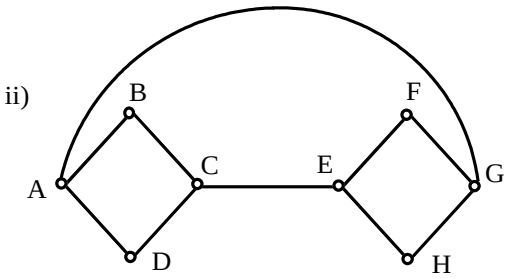
b) Prove that $C^n_{n-r} = C^n_r$ **4**

c) Determine Euler circuit for this graph. **4**

i)



ii)



d) Prove that $e'_1 = e''_1$, where e'_1 is a right identity and e''_1 is a left identity of a binary operation. **4**

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