B.Sc. (Information Technology)-I Second Semester Old

2BIT6 - Discrete Mathematics Paper - VI

P. Pages : 3 **GUG/W/18/1434**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions are compulsory and carry equal marks.
2. Draw neat and labelled diagram and use supporting data wherever necessary.
3. Avoid vague answers and write specific answer related to question.

1. Either

a) **8**

i) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ compute $A \cdot B$.

ii) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ compute $A \vee B$, $A \wedge B$

b) Prove that statement is true by using mathematical induction. **8**

i) $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2 = \dfrac{n(n+1)\,(2n+1)}{6}$

ii) $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

OR

c) i) To show $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ **8**

ii) $\sim (p \vee q) \equiv \sim p \wedge \sim q$ and
$\sim (p \wedge q) \equiv \sim P \vee \sim q$

d) Obtain the principal disjunctive normal form of **8**

i) $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$

ii) $\neg P \vee Q$

2. Either

a) Determine the value of the following. **8**

i) ${}^{4}P_{2}$ ii) ${}^{9}P_{3}$ iii) ${}^{20}P_{3}$ iv) ${}^{52}P_{4}$

b) Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 1), (2, 2) (2, 3), (2, 4), (3, 4), (4, 1)}. Draw diagraph and M_R of Relation. **8**

OR

c) Explain Warshall's algorithm. Let A = {1, 2, 3, 4} and B = {(2, 1), (2, 3), (3,2), (4, 3)} Find the transitive closure of R using Warshall's algorithm. **8**

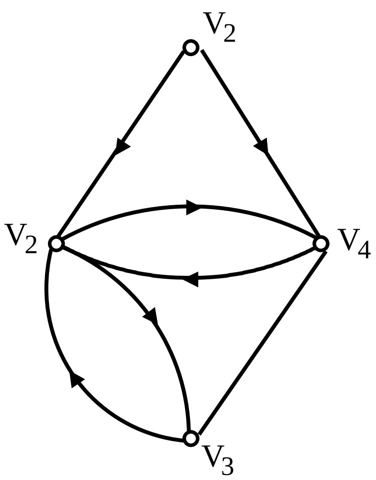
d) Let f, g, A be function from M to N where N is the set of natural nos so that f(n) = n+1, g(n) = 2n. **8**

$$h(n) = \begin{cases} 0 & \text{where n is even} \\ 1 & \text{where n is odd} \end{cases}$$

Determine $f \cdot f$, $f \cdot g$, $g \cdot f$, $h \cdot g$, $(f \cdot g) \cdot h$

3. Either

a) For the diagraph determine $A, A^1, A \cdot A^T, A^T \cdot A$ and $A \wedge A^T$ **8**



b) Define the following. **8**

i) Graph
ii) Adjacent node
iii) Diagraph
iv) Mixed Graph
v) Parallel Edges
vi) Loop
vii) Isolated graph
viii) Undirected graph

OR

c) Construct the tree **8**

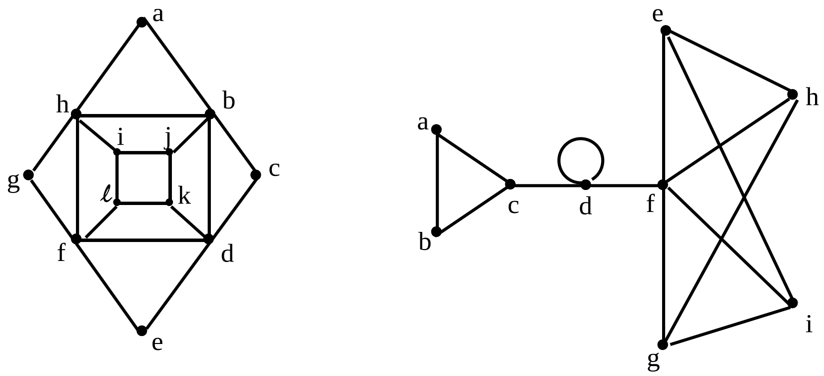
i) $((3*(1-n)) \div ((4+(7-(y+2))) * (7+(x \div y))$

ii) $3-(x+(6*(4 \div (2-3))))$

iii) $(7+(6-2)) - (x-(y-4))$

iv) $(x+(y-(x+y))) \times ((3 \div (2 \times 7)) \times 4)$

d) Find spanning tree for the graph. **8**



4. Either

a) Let T be the set of all even integers. Show that the semigroups (Z , +) and (T, +) are isomorphic. **8**

b) Let $(S, *)$ and $(T, *')$ be monoid with identities e and e' respectively. Let $f : S \to T$ be an isomorphism then $f(e) = e'$. **8**

OR

c) If N is a normal subgroup of G if and only if $g \bigcap g^{-1} = N$ **8**

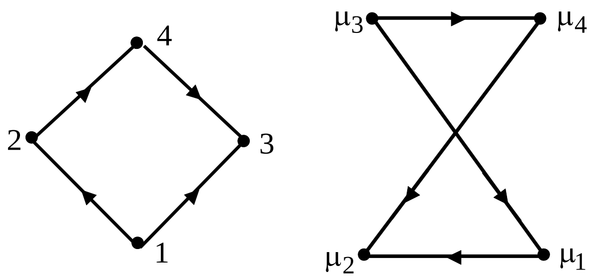
d) If N is a normal subgroup of G if and only if ${}_gN = N_g$. **8**

5. Solve all the questions.

a) Construct truth table for $\neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$ **4**

b) Let $X = \{1, 2, 3, 4\}$ and $R = \{(x, y) \mid x > y\}$. Draw diagraph of R and its matrix. **4**

c) Show that graph are isomorphic. **4**



d) A non empty subset H of the group G is a subgroup of G if and only if **4**

i) $a, b \in H \Rightarrow a \cdot b \in H$

ii) $a \in H \Rightarrow a^{-1} \in H$

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