B.Sc. First Year (CBCS Pattern) Sem-I

USMT-01 - Mathematics Paper-I : Differential and Integral Calculus

P. Pages : 2 **GUG/W/23/11556 (S)**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) If $\lim\limits_{x\to x_0} f(x)$ exists then prove that it is unique. **6**

b) Prove that $f(x)=x^2$ is continuous at x = 3 by $\in-\delta$ definition. **6**

OR

c) Discuss the continuity of the function **6**

$$f(x)=(x-a)\sin\frac{1}{x-a}, \quad x\neq a$$
$$=0 \qquad , x=a$$

d) If $y=\left(x+\sqrt{1+x^2}\right)^m$, then show that $(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$. **6**

UNIT – II

2. a) Verify Rolle's theorem for the function $f(x)=x^2+x-6$ in [-3 , 2]. **6**

b) If a real function f defined on [a, b] is (i) continuous on [a, b] (ii) differentiable on (a, b) then prove that there is at least one point $c\in(a,b)$ such that $f(b)-f(a)=(b-a)f'(c)$. **6**

OR

c) Show that $\frac{x}{1+x^2}<\tan^{-1}x<x, \forall x>0$. **6**

d) Obtain Maclaurin's series for the function $f(x)=\sin x$. **6**

UNIT – III

3. a) Prove that **6**

(i) $\boxed{n}=2\int_0^\infty e^{-t^2}t^{2n-1}dt$ (ii) $\boxed{n}=\int_0^1\left(\log\frac{1}{x}\right)^{n-1}dx$

b) Prove that $\beta(m,n)=\frac{\boxed{m}\,\boxed{n}}{\boxed{m+n}}$ **6**

OR

c) Prove that $\lim\limits_{x\to1}\left[\frac{1}{\log x}-\frac{x}{x-1}\right]=-\frac{1}{2}$. **6**

d) Prove that $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right) = 0$. **6**

UNIT – IV

4. a) Prove that **6**

i) $\iint_D c\, f(x, y)\,dA = c\iint_D f(x, y)\,dA$, C is constant.

ii) $\iint_D [f(x, y) + g(x, y)]\,dA = \iint_D f(x, y)\,dA + \iint g(x, y)\,dA$ where f(x, y), g(x, y) are continuous on D.

b) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$ **6**

OR

c) Evaluate $\iint_D \frac{dx\,dy}{x^4+y^2}$, where D is the region $x \ge 1,\ y \ge x^2$. **6**

d) Evaluate by changing to polar co-ordinates $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$. Hence show that **6**

$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.

5. Attempt **any six.**

a) Let $f(x) = \begin{cases} 2x-1, & x<0 \\ 2 & , x=0 \\ 2x+1, & x>0 \end{cases}$ **2**

Then find $\lim_{x\to 2} f(x)$, $\lim_{x\to 0+} f(x)$.

b) If $y = A\sin mx + B\cos mx$, prove that $y_2 + m^2 y = 0$. **2**

c) State Cauchy's mean value theorem. **2**

d) Write Taylor's theorem. **2**

e) Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$ **2**

f) Evaluate $\beta(5,3)$. **2**

g) Evaluate $\int_0^1 \int_1^3 xy^2\,dy\,dx$ **2**

h) If f(x, y), g(x, y) are continuous as D and f(x, y) $\le$ g(x, y) then prove that $\iint_D f(x, y)\,dA \le \iint_D g(x, y)\,dA$. **2**

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