Bachelor of Science (B.Sc.) (CBCS Pattern) First Semester

USMT-02 - Mathematics Paper-II (Differential Calculus and Trigonometry)

P. Pages : 2 **GUG/W/18/11557**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) Using $\in-\delta$ definition of Limit prove that $\lim_{(x,y)\to(1,1)}\left(x^2+2y\right)=3$. **6**

b) Let $z=f(x,y)=\dfrac{y\left(x^2+y^2\right)}{y^2+\left(x^2+y^2\right)^2}$ **6**

Show that f has limit 0 as $(x, y)\to(0, 0)$ on a ray x = at, y = bt, but f doesn't have limit 0 as $(x,y)\to(0,0)$ along $y=x^2$.

OR

c) If $u=\dfrac{e^{x+y+z}}{e^x+e^y+e^z}$, show that $u_x+u_y+u_z=2u$ where $u_x=\dfrac{\partial u}{\partial x}$ etc. **6**

d) If $u=F(x-y,\ y-z,\ z-x)$, then prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **6**

UNIT - II

2. a) Verify Euler's theorem as homogeneous functions for $u=\text{Log}\left(\dfrac{x+y}{x-y}\right)$. **6**

b) If $x=r\sin\theta\cos\phi$ $y=r\sin\theta\sin\phi$. $z=r\cos\theta$, find $\dfrac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$. **6**

OR

c) Expand x^3-2xy^2 in Taylor's series about the point (1, -1). **6**

d) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x-2y+2z=9$. **6**

UNIT - III

3. a) Find the asymptotes of the curve. **6**

$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$.

b) Find the curvature of the cycloid $x = \theta - \sin\theta,\ y = 1 - \cos\theta$ at the highest point of an arc. **6**

OR

c) Trace the curve: $a^2 y^2 = x^2\left(a^2 - y^2\right)$. **6**

d) Find the asymptotes of the curve $3x^2 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$. **6**

UNIT - IV

4. a) If n is a positive or negative integer, prove that **6**

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

b) Show that **6**

$(a+ib)^{m/n} + (a-ib)^{m/n} = 2\left(a^2+b^2\right)^{m/2n} \cdot \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$.

OR

c) Prove that $\tan h^{-1} x = \frac{1}{2}\,\mathrm{Log}\,\frac{1+x}{1-x}$. **6**

d) Show that **6**

$\mathrm{Log}(1+i) = \frac{1}{2}\log 2 + i\pi\left(2n + \frac{1}{4}\right)$.

5. Attempt **any six.**

a) Define $E - \delta$ definition of continuity of function f (x, y). **2**

b) If $u = x^2 + y^2$ where $x = at^2,\ y = 2at,$ find $\frac{du}{dt}$. **2**

c) State Euler's theorem for homogeneous function of two variables. **2**

d) If $x = r\cos\theta,\ y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and hence find $\frac{\partial(r,\theta)}{\partial(x,y)}$. **2**

e) Define Asymptote of a curve. **2**

f) Write the formula of radius of curvature of a curve in the cartesian form. **2**

g) Express $\left(i + i\sqrt{3}\right)$ in polar form. **2**

h) Separate cos h (x+iy) into real and imaginary parts. **2**

****