B.Sc. - I (With Credits)-Regular-Semester 2012 Sem I

MAT-102 - Mathematics Paper-II (Differential and Integral Calculus)

P. Pages : 3 **GUG/S/18/3318**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. Q. 1 to 4 have an alternative solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT - I

1. a) Show that the function **6**

$$f(x) = (1+2x)^{1/x}, \quad x \neq 0$$
$$= e^2 \quad x = 0$$

is continuous at x = 0.

b) Let f(x) and g(x) be defined at all. Points of an interval [a, b] except possibley at $x_0 \in [a, b]$ **6**

if $\lim_{x \to x_0} f(x) = A$, $\lim_{x \to x_0} g(x) = B$ then prove that

$$\lim_{x \to x_0} \{f(x) + g(x)\} = \lim_{x \to x_0} f(x) + \lim_{x \to x_0} g(x) = A + B$$

OR

c) Verify Lagrange's mean value theorem for the function **6**

$f(x) = 2x^2 - 7x + 10$ in [2, 5]

d) If f and g are continuous real functions on [a, b] which are differentiable table in (a, b) **6**

then there is a point $C \in (a, b)$ such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(C)}{g'(C)}$$

UNIT - II

2. a) If $y^3 + 3ax^2 + x^3 = 0$ then show that $y^5 y_2 + 2a^2 x^2 = 0$. **6**

b) If $y = \left(x + \sqrt{1 + x^2}\right)^m$ then show that **6**

i) $\left(1 + x^2\right) y_2 + x y_1 - m^2 y = 0$ and

ii) $\left(1 + x^2\right) y_{n+2} + (2n+1) x \cdot y_{n+1} + \left(n^2 - m^2\right) \cdot y_n = 0$

OR

c) Evaluate the following limit **6**

$$\lim_{x\to 0} (\cot x)^{\sin 2x}$$

d) Prove that $\lim_{x\to 0} \log_{\tan x} \tan 2x = 1$ **6**

UNIT - III

3. a) Evaluate $\int \frac{x+1}{\sqrt{x^2 - x + 1}} dx$ **6**

b) Evaluate $\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}} dx$ **6**

OR

c) If $\phi(n) = \int_0^{\pi/4} \tan^n x \cdot dx$ **6**

then show that

$\phi(n) + \phi(n-2) = \frac{1}{n-1}$ then find the value of $\phi(s)$.

d) If $In = \int \sec^n x \cdot dx$ then prove that **6**

$$In = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} In - 2$$

UNIT - IV

4. a) Prove that $\left\lceil 1/2 \right. = \sqrt{\pi}$. **6**

b) Prove that **6**

$$B(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta .$$

OR

c) Prove that **6**

$$\beta(m, n) = \frac{\left\lceil m \right. \left\lceil n \right.}{\left\lceil m+n \right.}$$

d) Prove that **6**

$$\int_0^{\pi/2} \sqrt{\tan\theta} \cdot d\theta = \frac{\pi}{\sqrt{2}}$$

5. Solve **any six.**

a) Evaluate $\lim_{x \to 2} \frac{(x+1)(3x-4)}{(x-1)(2x-3)}$ **2**

b) Show that $f(x) = \frac{1}{1-e^{1/x}}$ has a simple discontinuity at x = 0. **2**

c) If $y = \frac{1}{ax+b}$ then prove that $y_n = (-1)^n n!\, a^n (ax+b)^{-n-1}$ **2**

d) $y = \sin(1-x)$ then find y_4. **2**

e) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$ **2**

f) Integrate $\sqrt{x^2 + 2x + 5}$ **2**

g) Evaluate $\beta\left(\frac{3}{2}, \frac{5}{2}\right)$ **2**

h) Prove that $\int_0^{\pi/2} \sin^2\theta \cdot \cos^4\theta \cdot d\theta = \frac{\pi}{32}$ **2**

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