B.Sc. - I Sem-I (Old Course)

MAT-102 - Mathematics Paper-II
(Differential and Integral Calculus)

P. Pages : 2 **GUG/S/19/1215**

Time : Three Hours * 0 9 1 3 * Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) If $\lim_{x \to x_0} f(x)$ exists, then prove that it is unique. **6**

b) **6**

If $f(x) = \frac{e^{1/x}}{1+e^{1/x}}$, $x \neq 0$

$= 0$, $x = 0$

show that f(x) has a simple discontinuity at x = 0.

OR

c) Show that $\frac{x}{1+x^2} < \tan^{-1} x < x, \forall x > 0$. **6**

d) Expand $2x^3 + 7x^2 + x - 1$ in powers of (x - z) in Taylor's series form. **6**

UNIT - II

2. a) Find the n^{th} differential coefficient of $\frac{1}{1-5x+6x^2}$ **6**

b) If $p^2 = a^2 \cos^2\theta + b^2 \sin^2\theta$ then prove that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$ **6**

OR

c) Evaluate $\lim_{x \to 0} \left(\cot x - \frac{1}{x}\right)$ **6**

d) Evaluate $\lim_{x \to 0} (\cos x)^{1/x^2}$ **6**

UNIT - III

3. a) Evaluate $\int \frac{x+1}{\sqrt{x^2-x+1}} dx$ **6**

b) Evaluate $\int (2+5)\sqrt{x^2+3x+1}\, dx$ **6**

OR

c) If $I_n = \int \sec^n x\,dx$ then prove that $I_n = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}$ **6**

d) Evaluate the Integral $\int \frac{dx}{(x^2+1)^2}$ **6**

UNIT - IV

4. a) Show that $\Gamma(n+1) = n\,\Gamma(n)$. **6**

b) Prove that $\int_0^{\infty} e^{-h^2x^2} dx = \frac{\sqrt{\pi}}{2h}$ **6**

OR

c) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$ **6**

d) Prove that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1\,;\, n+1)$ **6**

5. Solve **any six.**

a) $\lim_{x \to 0} \frac{1}{|x|} = \infty$ **2**

b) Write Maclaurin's series for f(x) at $x_0 = 0$. **2**

c) If $y = x^2 e^x$, show that $y_n = \frac{1}{2} n(n-1) y_2 - n(n-2) y_1 + \frac{1}{2}(n-1)(n-2) y$ **2**

d) Find y_3, If $y = (2x-3)^4$. **2**

e) Integrate $\int \sqrt{3 + 4x - 4x^2}\, dx$ **2**

f) Evaluate $\int_0^{\pi/4} \sin^4 x\, dx$ **2**

g) Evaluate $\int_0^{\infty} x^{1/4}\, e^{-\sqrt{x}}\, dx$ **2**

h) Show that $\int_0^{\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$ **2**

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