B.Sc. (C.B.C.S. Pattern) Sem-I

USMT-01 - Mathematics Paper-I
(Differential and Integral Calculus)

P. Pages : 3 **GUG/S/19/11556**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that if $\lim_{x \to x_0} f(x)$ exists, then it is unique. **6**

b) Show that the function f defined by **6**

$$f(x) = x \sin\frac{1}{x}, \quad x \neq 0$$
$$= 0 \quad \text{otherwise}$$

is continuous at x = 0.

OR

c) If f(x) is differentiable at $x = x_0$, then prove that it is continuous at x_0. **6**

d) If $y = \left(x + \sqrt{1+x^2}\right)^m$, then show that **6**

i) $(1+x^2)y_2 + xy_1 - m^2 y = 0$

ii) $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

UNIT - II

2. a) Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$ in [2, 5]. **6**

b) If f and g are continuous real function on [a, b] which are differentiable in (a, b) then prove that there is point $c \in (a, b)$ **6**

such that $\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$

where $g(a) \neq g(b)$ and $f'(x), g'(x)$ are not simultaneously zero.

OR

c) Write Taylor's series for $f(x) = (1-x)^{5/2}$ with Lagrange's form of remainder up to three terms in the interval [0, 1] **6**

d) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - z)$. **6**

UNIT - III

3. a) Prove that - **6**

i) $\int_0^\infty x^{n-1} e^{-kx}\, dx = \frac{\Gamma n}{k^n}$ where n, k > 0 are constants.

ii) Evaluate $\int_0^\infty x^{1/4} e^{-\sqrt{x}}\, dx$

b) Prove that $\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$ **6**

OR

c) If f(x) and g(x) are differentiable in the interval (a, b) except possibly at a point $x_0 \in (a, b)$ and $f(x_0) = g(x_0) = 0$, then prove that $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$, if limit exists. **6**

d) Prove that $\lim_{x \to 0} \left(\cot x - \frac{1}{x} \right) = 0$ **6**

UNIT - IV

4. a) If f(x, y) , g(x, y) are continuous on region D then prove that - **6**

i) $f(x, y) \geq 0 \Rightarrow \iint_D f(x, y)\, dA \geq 0$ on D.

ii) $f(x, y) \leq g(x, y) \Rightarrow \iint_D f(x, y)\, dA \leq \iint_D g(x, y)\, dA$

b) Evaluate $\int_{-2}^{2} dy \int_{y^2-1}^{3} (x + 2y)\, dx$ **6**

OR

c) Evaluate $\iint_D r^2 \cdot \sin\theta \; dr \cdot d\theta$, where D is the semicircle $r = 2a\cos\theta$ above the initial line. **6**

d) Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$ **6**

5. Solve **any six.**

a) If $f(x)=\begin{cases} x^2 & , \quad x \neq 3 \\ 2 & , \quad x = 3 \end{cases}$ then show that f(x) is discontinuous at x = 3. **2**

b) Let f $f(x)=|x|, \forall x \in R$, then show that f(x) is not derivable at x = 0. **2**

c) State the Lagrange's mean value theorem for a real function defined on [a, b]. **2**

d) Define a power series in x and (x - a) **2**

e) Find $B\left(\frac{3}{2}, \frac{5}{2}\right)$ **2**

f) Evaluate **2**

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan 5x}{\tan x}$$

g) If f(x, y) be the function defined on region D of area A, then prove that **2**
$\iint\limits_D C f(x, y) dA = C \iint\limits_D f(x, y) dA$, where C is constant.

h) Change the order of integration **2**

$$\int_0^2 \int_{-1}^{1-y} x^{1/3} y^{-1/2} (1-x-y)^{1/2} dx \cdot dy$$

****