B.E. Mechanical Engineering Third Semester (Old)

ME301 - Applied Mathematics - III

P. Pages : 2 **GUG/W/18/1513**

Time : Three Hours Max. Marks : 80

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Notes :
1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) i) Find $L\left\{(t^2-1)\sin t\right\}$. **4**

ii) Find Laplace transform of $\dfrac{\cos at-\cos bt}{t}$ hence evaluate $\displaystyle\int_0^\infty \left\{\frac{\cos at-\cos bt}{t}\right\}dt$ **5**

b) Express $f(t)=\begin{cases}\cos t & , \quad 0<t<\pi \\ \cos 2t & , \quad \pi<t<2\pi \\ \cos 3t & , \quad t>2\pi\end{cases}$ **7**

in terms of unit step function and hence find its Laplace transform.

OR

2. a) Use convolution theorem to find $L^{-1}\left\{\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **8**

b) Find y(t) using Laplace Transform if $\dfrac{dy}{dt}+2y+\displaystyle\int_0^t y\,dt=\sin t$ given y(0) = 1. **8**

3. a) Find A^{-1} by Partitioning if $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$ **8**

b) Find the value of λ for which the following equations are consistent. Also solve the system for these value of λ **8**

$x+2y+z=3$

$x+y+z=\lambda$

$3x+y+3z=\lambda^2$

OR

4. a) Find the matrix which diagonalize the matrix $A=\begin{bmatrix}-1 & 1 & 2\\ 0 & -2 & 1\\ 0 & 0 & -3\end{bmatrix}$ hence find diagonal matrix. **8**

b) Find the rank of matrix $\begin{bmatrix}5 & 3 & 14 & 4\\ 0 & 1 & 2 & 1\\ 1 & -1 & 2 & 0\end{bmatrix}$ **4**

c) Investigate whether the vectors $x_1=(1, 2, 4)$ $x_2=(2, -1, 3)$ $x_3=(0, 1, 2)$, $x_4=(-3, 7, 2)$ are linearly dependent or not. **4**

Also find relation between them.

5\. a) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ then show that $\operatorname{cosec}^2 A - \cot^2 A = I$ by Sylvester's theorem. **8**

b) Verify Cayley-Hamilton theorem for given matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ hence find **8**

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

OR

6\. a) Solve $\frac{d^2y}{dt^2} + 4y = 0$ given $y = 8, \frac{dy}{dt} = 0$ when t = 0 by matrix method. **8**

b) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 + 4x_1x_3 - 8x_2x_3$ to cannocial form by an orthogonal transformation. **8**

7\. a) Solve $z(p-q) = z^2 + (x+y)^2$ **4**

b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ **4**

c) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \tan(x+y) + e^{x+y}$ **8**

OR

8\. a) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x \sin y$ **8**

b) Solve using method of separation of variable $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = \mu$, given that **8**

$\mu(x, 0) = 3e^{-5x} + 2e^{-3x}$

9\. a) Find the Fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$ **8**

hence show that $f(x) = \frac{8}{\pi^2}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...\right]$

b) Find half range cosine series for $f(x) = (x-1)^2$, $0 < x < 1$. **8**

OR

10\. a) Using Fourier integrals show that $\int_0^\infty \frac{(1-\cos \pi\lambda)\sin \lambda x}{\lambda} d\lambda = \begin{cases} \pi/2 & 0 < x < \pi \\ 0 & x > \pi \end{cases}$ **8**

b) Solve the integral equation $\int_0^\infty f(x) \cos \alpha x \, dx = \begin{cases} 1-\alpha & 0 \le \alpha \le 1 \\ 0 & \alpha > 1 \end{cases}$ **8**

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