

ASH3014 - Applied Mathematics-III / Mathematics-III

P. Pages : 3

Time : Three Hours



GUG/S/18/3714

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Use Laplace transform to evaluate the integral. 8

$$\int_0^{\infty} \frac{e^{-t} \sin^2 t}{t} dt$$

- b) Find the Laplace transform of $f(t)$ where 8

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}. \end{cases}$$

and $f(t + \frac{2\pi}{\omega}) = f(t)$.

OR

2. a) Use convolution theorem to find. 8

$$L^{-1} \left\{ \frac{s^2}{(s^2 + 4)^2} \right\}.$$

- b) Solve, by Laplace transform method 8

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 2(1+t-t^2)$$

given that $y(0)=0$ and $y'(0)=3$.

3. a) Find the inverse of a matrix. 8

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

by the method of partitioning.

- b) Find the eigen values, eigen vectors and the modal matrix for the matrix. 8

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

OR

4. a) Test for consistency and solve. 8

$$2x - y - z = 2$$

$$x + 2y + z = 2$$

$$4x - 7y - 5z = 2$$

- b) Are vectors. 4

$$X_1 = (1, 1, 1, 3), X_2 = (1, 2, 3, 4), X_3 = (2, 3, 4, 9)$$

linearly dependent? If so, find the relation between them.

- c) If $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 7 & 4 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ 4

$$\text{and } \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Express $x_1 + x_2 + x_3$ in terms of z_1 and z_2 .

5. a) Verify Caley-Hamilton's theorem for the matrix. 8

$$\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

and hence find it's inverse.

- b) Use Sylvester's theorem to show that 8

$$2\sin A = (\sin 2)A$$

$$\text{where } A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}.$$

OR

6. a) Solve by matrix method 8

$$\frac{d^2y}{dt^2} - 5 \frac{dy}{dt} + by = 0$$

given that $y(0) = 2, \frac{dy}{dt} = 5$ at $t = 0$.

- b) Find the largest eigen value and the corresponding eigen vector of the matrix. 8

$$\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

7. a) Solve 4

$$y^2 p - xyq = x(z - 2y).$$

- b) Solve 4

$$x(y - z)p + y(z - x)q = z(x - y).$$

c) Solve

$$\frac{\partial^2 z}{\partial x^2} - 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = e^{2x+3y} + \sin(x-2y).$$

OR

8. a) Solve

$$\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} - 8 \frac{\partial^2 z}{\partial y^2} = e^{2x+y} + \sqrt{2x+3y}.$$

b) Solve, by the method of separation of variables.

$$\frac{\partial u}{\partial x} - 2 \frac{\partial u}{\partial y} = u$$

$$\text{given that } u(x,0) = 3e^{-5x} + 2e^{-3x}.$$

9. a) Obtain the Fourier Series for

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$$

Hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

b) Find half-range cosine series for $f(x) = \sin x$, $0 < x < \pi$.

OR

10. a) Show that the Fourier sine integral of

$$f(x) = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

$$\text{is } \int_0^\infty \frac{(1-\cos \pi \lambda) \sin \lambda x}{\lambda} d\lambda.$$

b) Solve, the integral equation.

$$\int_0^\infty f(t) \cos \lambda t dt = \begin{cases} 1, & 0 \leq \lambda < 1 \\ 2, & 1 \leq \lambda < 2 \\ 0, & \lambda > 2 \end{cases}$$
