B.E.(with Credits)-Regular-Semester 2012- All Branches Sem I & II (Old Pattern)

105 - Applied Mathematics-I

P. Pages : 3 **GUG/S/18/3665**

Time : Three Hours Max. Marks : 80

* 0 8 4 0 *

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Find the n^{th} differential coefficient of $\cos 2x \cdot \cos 3x$. **3**

b) If $y = x \log\left(\frac{x-1}{x+1}\right)$ Then prove that $\frac{d^n y}{dx^n} = (-1)^n (n-2)! \left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$. **7**

c) If $y = \sin\left\{\log\left(x^2 + 2x + 1\right)\right\}$ Then show that **6**

$(x+1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + \left(n^2 + 4\right) y_n = 0$.

OR

2. a) If $y = \sin\left(2\sin^{-1} x\right)$ Then prove that $\left(1-x^2\right) y_{n+2} - (2n+1) x\, y_{n+1} - \left(n^2 - 4\right) y_n = 0$ **8**

Hence or otherwise show that $\sin\left(2\sin^{-1} x\right) = 2x - x^3 - \frac{x^5}{5} ----$

b) Expand $2x^3 + 7x^2 + x - 2$ in powers of (x – 2) by using Taylor's theorem. **4**

c) Evaluate $\lim_{x \to 0} \frac{x e^x - \log(1+x)}{x \sin x}$. **4**

3. a) If $u = x \log(x+r) - r$ where $r^2 = x^2 + y^2$ **8**

Then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{x+r}$.

b) If $u = \tan^{-1}\left(y^2 / x\right)$ **8**

Then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$.

OR

4. a) If $u = \frac{\left(x^2 + y^2\right)^n}{2n(2n-1)} + x f(y/x)$ Then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \left(x^2 + y^2\right)^n$. **8**

b) If $u = f(x, y)$ where $x = s\cos\alpha - t\sin\alpha$ and $y = s\sin\alpha + t\cos\alpha$ **8**

Then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}$.

5. a) If $x = r\cos\theta$ and $y = r\sin\theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$ and $\frac{\partial(r, \theta)}{\partial(x, y)}$ and varify that **8**

$\frac{\partial(x, y)}{\partial(r, \theta)} \times \frac{\partial(r, \theta)}{\partial(x, y)} = 1$.

b) Expand $e^x \log(1 + y)$ in powers of x and y as far as term containing third degree by using Maclaurin's theorem. **8**

OR

6. a) If $u = \frac{x + y}{x - y}$ and $v = \frac{xy}{(x - y)^2}$. **8**

Are u and v functionally related? If so, find the relation between them.

b) Show that the stationary value of $u = x^m y^n z^p$ where $x + y + z = a$ is **8**

$m^m n^n p^p \left(\frac{a}{m + n + p}\right)^{m+n+p}$

by using Langrange's method of undetermined multipliers.

7. a) By differentiating under the integral sign **8**

Evaluate $\int_0^\infty \frac{e^{-ax} \sin x}{x} dx$

Hence show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$

b) Evaluate $\int_0^\infty \sqrt{x}\, e^{-\sqrt[3]{x}} dx$ **4**

c) Evaluate $\int_0^a x^4 \sqrt{a^2 - x^2}\, dx$ **4**

OR

8. a) Trace the curve $xy^2 = 4a^2(2a - x)$. **8**

Also find the area included between the curve and its asymptote.

b) Find the root mean square value of $f(x) = \log x$ over the range x = 1 to x = e, substitute the value of e in the result and evaluate it further. **8**

9. a) Fit a second degree parabola $y = a + bx + cx^2$ for the following data. **8**

x	20	30	40	50	60
y	54	90	138	206	292

b) Calculate the coefficient of correlation and equations to the lines of regressions for the following data. **8**

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

OR

10. a) Fit a curve of the type $y = ab^x$ to the following data. **8**

x	1	2	3	4	5	6
y	1.6	4.5	13.8	40.2	125	300

b) Calculate the rank correlation coefficient for the following data. **8**

x	60	34	40	50	45	40	22	43	42	66	64	46
y	75	32	33	40	45	30	12	30	34	72	41	57

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